Adaptive Impact Absorption (AIA'15)
IPPT PAN, Warsaw
15-16 October 2015

Pole Placement and Active Vibration Control in Aeroelasticity

John E Mottershead[1], Yitshak Ram[2], Shakir Jiffri[1], Sabastiano Fichera[1] and Xiaojun Wei[1]

[1]*University of Liverpool UK*

[2]*Louisiana State University, Baton Rouge, USA*

j.e.mottershead@liv.ac.uk

UNIVERSITY OF LIVERPOOL

EPSRC
Engineering and Physical Sciences Research Council

Contents

Receptance Method

- Dynamic stiffnesses $\rightarrow$ Receptances:

$$\left(s^2\mathbf{M}+s\mathbf{C}+\mathbf{K}\right)\mathbf{x}(s)=\mathbf{f}(s)$$

$$\mathbf{H}(s)=\left(s^2\mathbf{M}+s\mathbf{C}+\mathbf{K}\right)^{-1}$$

$$\mathbf{H}(s)\mathbf{f}(s)=\mathbf{x}(s)$$

- No need to evaluate or to know the system matrices **M**, **C**, **K**.
- Any input-output transfer function may be used – dynamics of actuators, sensors, filters etc. included in the measurement.

Receptance Method
Partial Pole Placement Problem

Open–loop and closed-loop systems:

$$\left.\begin{aligned} \lambda_k^2\mathbf{M}+\lambda_k\mathbf{C}+\mathbf{K}\;\;\mathbf{v}_k &= \mathbf{0} \\[2em] \mu_k^2\mathbf{M}+\mu_k\mathbf{C}+\mathbf{K}\;\;\mathbf{w}_k &= \mathbf{Bu}(t) \\ \mathbf{u}(t) = \;\; \mu_k\mathbf{F}^T+\mathbf{G}^T\;\;\mathbf{w}_k & \end{aligned}\right\} k=1,2,\ldots,2n$$

Assigned eigenvalues $\mu_k \;{}_{k=1}^{p}$ are distinct from eigenvalues $\lambda_k \;{}_{k=1}^{2n}$

While eigenvalues $\mu_k=\lambda_k \quad k=p+1, p+2,\ldots,2n$ are unchanged.

Unchanged Eigenvalues

$$\left(\lambda_k^2\mathbf{M}+\lambda_k\mathbf{C}+\mathbf{K}\right)\mathbf{w}_k=\mathbf{B}\left(\lambda_k\mathbf{F}^T+\mathbf{G}^T\right)\mathbf{w}_k$$

$$\mathbf{B}=\begin{bmatrix}\mathbf{b}_1 & \mathbf{b}_2 & \cdots & \mathbf{b}_m\end{bmatrix}\qquad \mathbf{F}=\begin{bmatrix}\mathbf{f}_1 & \mathbf{f}_2 & \cdots & \mathbf{f}_m\end{bmatrix}\qquad \mathbf{G}=\begin{bmatrix}\mathbf{g}_1 & \mathbf{g}_2 & \cdots & \mathbf{g}_m\end{bmatrix}$$

May be re-written as,

$$\left(\mu_k^2\mathbf{M}+\mu_k\mathbf{C}+\mathbf{K}\right)\mathbf{w}_k=\left(\mathbf{b}_1\left(\mu_k\mathbf{f}_1^T+\mathbf{g}_1^T\right)+\mathbf{b}_2\left(\mu_k\mathbf{f}_2^T+\mathbf{g}_2^T\right)+\ldots+\mathbf{b}_m\left(\mu_k\mathbf{f}_m^T+\mathbf{g}_m^T\right)\right)\mathbf{w}_k$$

Non-trivial solution: $\mathbf{w}_k=\mathbf{v}_k$

$$\left(\mathbf{b}_1\left(\lambda_k\mathbf{f}_1^T+\mathbf{g}_1^T\right)+\ldots+\mathbf{b}_m\left(\lambda_k\mathbf{f}_m^T+\mathbf{g}_m^T\right)\right)\mathbf{v}_k=\mathbf{0}\qquad k=p+1,p+2,\ldots,2n$$

$$\begin{bmatrix}\lambda_k\mathbf{v}_k^T & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{v}_k^T & \mathbf{0} & \cdots & \mathbf{0}\\ \mathbf{0} & \lambda_k\mathbf{v}_k^T & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{v}_k^T & \cdots & \mathbf{0}\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ \mathbf{0} & \mathbf{0} & \cdots & \lambda_k\mathbf{v}_k^T & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{v}_k^T\end{bmatrix}\begin{pmatrix}\mathbf{f}_1\\ \vdots\\ \mathbf{f}_m\\ \mathbf{g}_1\\ \vdots\\ \mathbf{g}_m\end{pmatrix}=\begin{pmatrix}0\\ 0\\ \vdots\\ 0\end{pmatrix}$$

Assigned Eigenvalues

$$\mathbf{w}_k = \mathbf{H}(\mu_k)\left[\mathbf{b}_1\left(\mu_k \mathbf{f}_1^T + \mathbf{g}_1^T\right) + \mathbf{b}_2\left(\mu_k \mathbf{f}_2^T + \mathbf{g}_2^T\right) + \ldots + \mathbf{b}_m\left(\mu_k \mathbf{f}_m^T + \mathbf{g}_m^T\right)\right]\mathbf{w}_k \qquad k = 1,2,\ldots,p$$

Let $\quad \mathbf{r}_{\mu_k,j} = \mathbf{H}(\mu_k)\mathbf{b}_j \quad$ and $\quad \alpha_{\mu_k,j} = \left(\mu_k \mathbf{f}_j^T + \mathbf{g}_j^T\right)\mathbf{w}_k \qquad j = 1,2,\ldots,m$

Then

$$\begin{bmatrix} \mu_k \mathbf{w}_k^T & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{w}_k^T & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mu_k \mathbf{w}_k^T & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{w}_k^T & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mu_k \mathbf{w}_k^T & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{w}_k^T \end{bmatrix} \begin{pmatrix} \mathbf{f}_1 \\ \vdots \\ \mathbf{f}_m \\ \mathbf{g}_1 \\ \vdots \\ \mathbf{g}_m \end{pmatrix} = \begin{pmatrix} \alpha_{\mu_k,1} \\ \alpha_{\mu_k,2} \\ \vdots \\ \alpha_{\mu_k,m} \end{pmatrix}$$

where $\quad \mathbf{w}_k = \alpha_{\mu_k,1}\mathbf{r}_{\mu_k,1} + \alpha_{\mu_k,2}\mathbf{r}_{\mu_k,2} + \ldots + \alpha_{\mu_k,m}\mathbf{r}_{\mu_k,m}$

The closed-loop mode shape is determined by the choice of $\alpha_{\mu_k,j}$

Receptance Method

The control gains are then given by the solution of,

$$\begin{bmatrix} \mathbf{P}_1 \\ \vdots \\ \mathbf{P}_p \\ \mathbf{Q}_{p+1} \\ \vdots \\ \mathbf{Q}_{2n} \end{bmatrix} \begin{pmatrix} \mathbf{f}_1 \\ \vdots \\ \mathbf{f}_m \\ \mathbf{g}_1 \\ \vdots \\ \mathbf{g}_m \end{pmatrix} = \begin{pmatrix} \boldsymbol{\alpha}_1 \\ \vdots \\ \boldsymbol{\alpha}_\mathbf{p} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{pmatrix}$$

$$\mathbf{P}_k = \begin{bmatrix} \mu_k \mathbf{w}_k^T & 0 & \cdots & 0 & \mathbf{w}_k^T & 0 & \cdots & 0 \\ 0 & \mu_k \mathbf{w}_k^T & \cdots & 0 & 0 & \mathbf{w}_k^T & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mu_k \mathbf{w}_k^T & 0 & 0 & \cdots & \mathbf{w}_k^T \end{bmatrix}$$

$$\mathbf{Q}_k = \begin{bmatrix} \lambda_k \mathbf{v}_k^T & 0 & \cdots & 0 & \mathbf{v}_k^T & 0 & \cdots & 0 \\ 0 & \lambda_k \mathbf{v}_k^T & \cdots & 0 & 0 & \mathbf{v}_k^T & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_k \mathbf{v}_k^T & 0 & 0 & \cdots & \mathbf{v}_k^T \end{bmatrix}$$

Y.M. Ram and J.E. Mottershead, Multiple-input active vibration control by partial pole placement using the method of receptances, *Mechanical Systems and Signal Processing*, 40, 2013, 727-735.

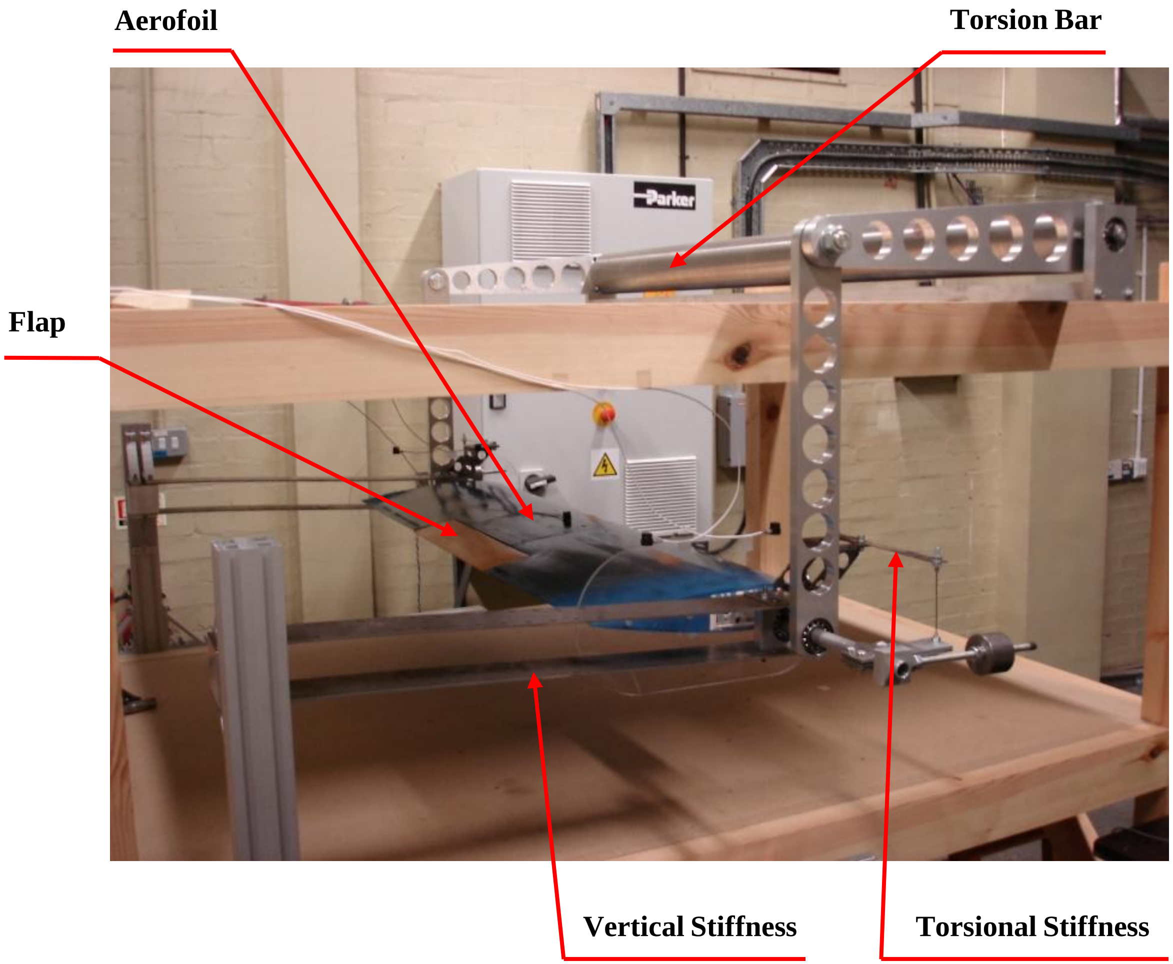
General Procedure

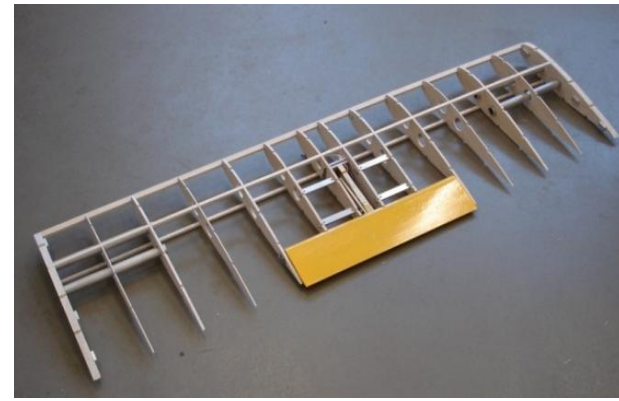
- Measure the open loop input-output FRF over a desired frequency range.
- Fit MIMO rational fraction polynomials to the measure FRF and obtain the input-output transfer functions.
- Select force distribution vectors $\mathbf{b}_k(s)$.
- Apply the Receptance Method to obtain unknown gains, $\mathbf{g}_k$, $\mathbf{f}_k$.

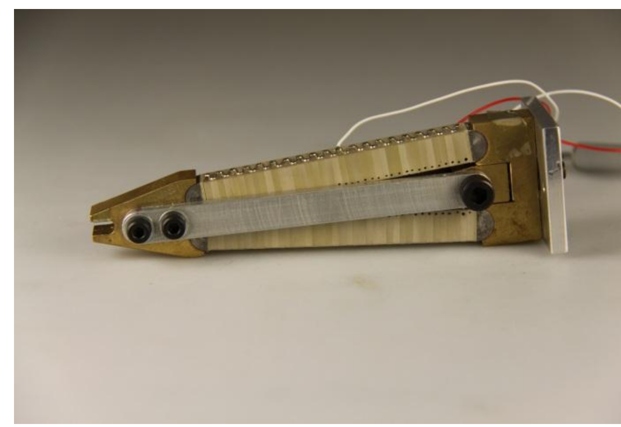
$$\begin{bmatrix} \mathbf{P}_1 \\ \vdots \\ \mathbf{P}_p \\ \mathbf{Q}_{p+1} \\ \vdots \\ \mathbf{Q}_{2n} \end{bmatrix} \begin{pmatrix} \mathbf{f}_1 \\ \vdots \\ \mathbf{f}_m \\ \mathbf{g}_1 \\ \vdots \\ \mathbf{g}_m \end{pmatrix} = \begin{pmatrix} \boldsymbol{\alpha}_\mathbf{1} \\ \vdots \\ \boldsymbol{\alpha}_\mathbf{p} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{pmatrix}$$

- Implementation of the controller using dSPACE in real time.

Flutter Suppression
Wind-Tunnel Aerofoil Rig

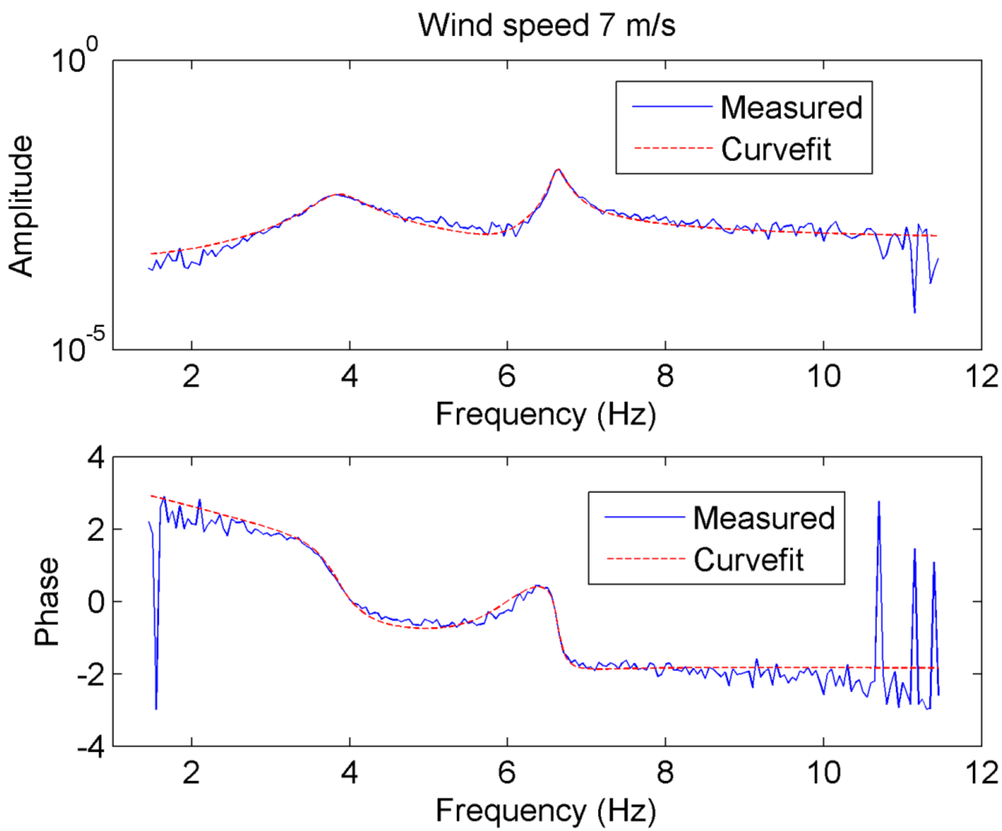






V-stack piezo actuator

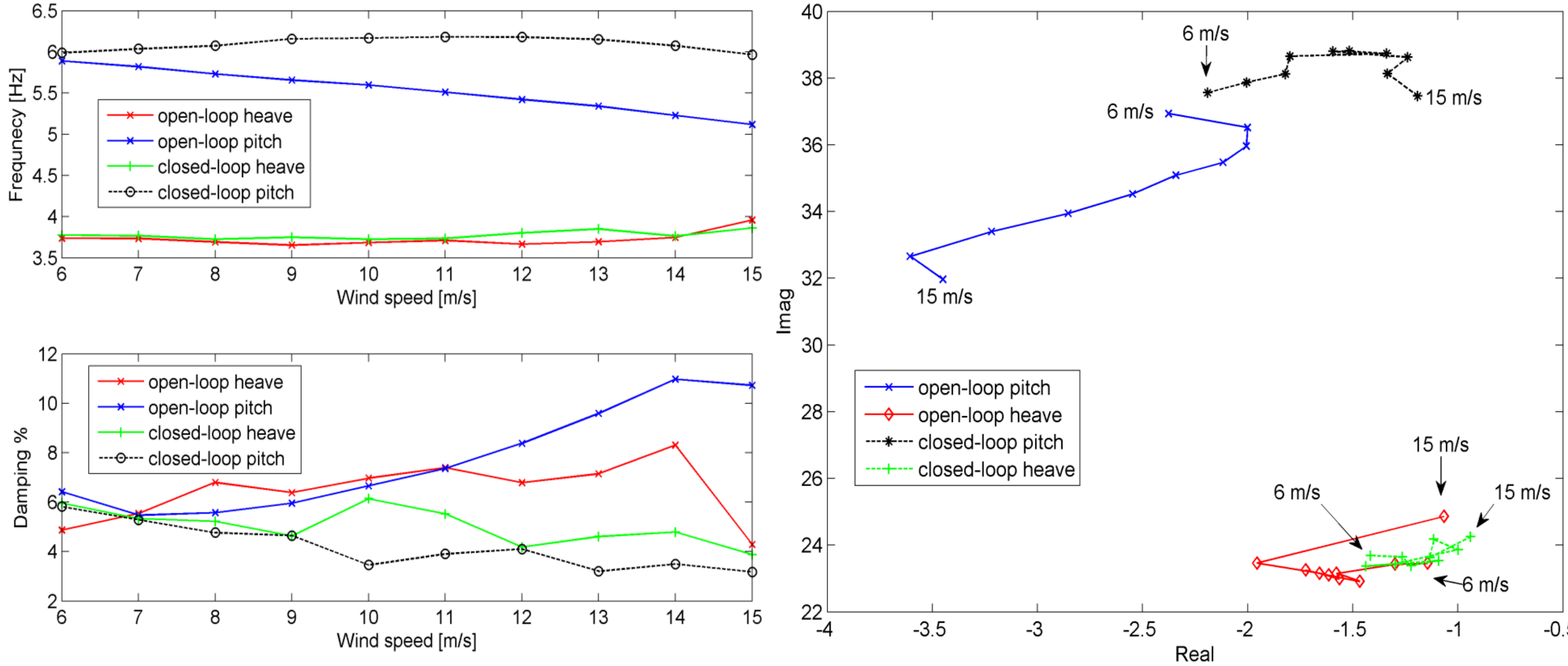
Curve Fitting



Open loop FRFs include not only the dynamics of the aerofoil system but also the power amplifier, the actuator, the sensors and the effects of A/D and D/A conversion, numerical differentiation (Simulink/dSPACE) of displacements and high- and low-pass Butterworth filters with cut-off frequencies of 1Hz and 35 Hz.

Frequency/Damping Trends – Root Locus

Separation of pitch and heave frequencies



Poles assigned: $\mu_{pitch} = -1.5 \pm 38i, \quad \mu_{heave} = -0.7 \pm 23i$

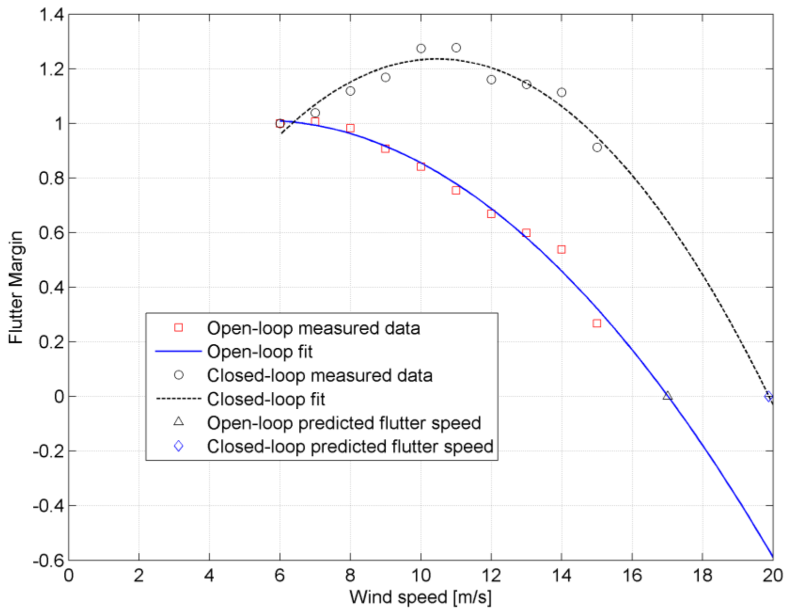
at wind speed, 7m/s.

Flutter Margin

$$F = \left[\left(\frac{\tilde{\omega}_2^2 - \tilde{\omega}_1^2}{2}\right) + \left(\frac{\sigma_2^2 - \sigma_1^2}{2}\right)\right]^2 + 4\sigma_1\sigma_2\left[\left(\frac{\tilde{\omega}_2^2 + \tilde{\omega}_1^2}{2}\right) + 2\left(\frac{\sigma_2^2 + \sigma_1^2}{2}\right)^2\right]$$
$$- \left[\left(\frac{\sigma_2 - \sigma_1}{\sigma_2 + \sigma_1}\right)\left(\frac{\tilde{\omega}_2^2 - \tilde{\omega}_1^2}{2}\right) + 2\left(\frac{\sigma_2 + \sigma_1}{2}\right)^2\right]^2$$

$$\tilde{\omega}_j = \omega_j\sqrt{(1 - \zeta_j^2)}$$

$$\sigma_j = -\zeta_j\omega_j \qquad j = 1{,}2$$



Zimmerman N.H. and Weissenburger J.T. (1964)

Quadratic flutter speed prediction.

Predicted flutter speed increased from 17 m/s to 20 m/s.

Separation of pitch and heave frequencies.

$$\mathbf{g} = -28.2 \quad 29.5^{T} \qquad \mathbf{f} = 0.0284 \quad 0.0232^{T}$$

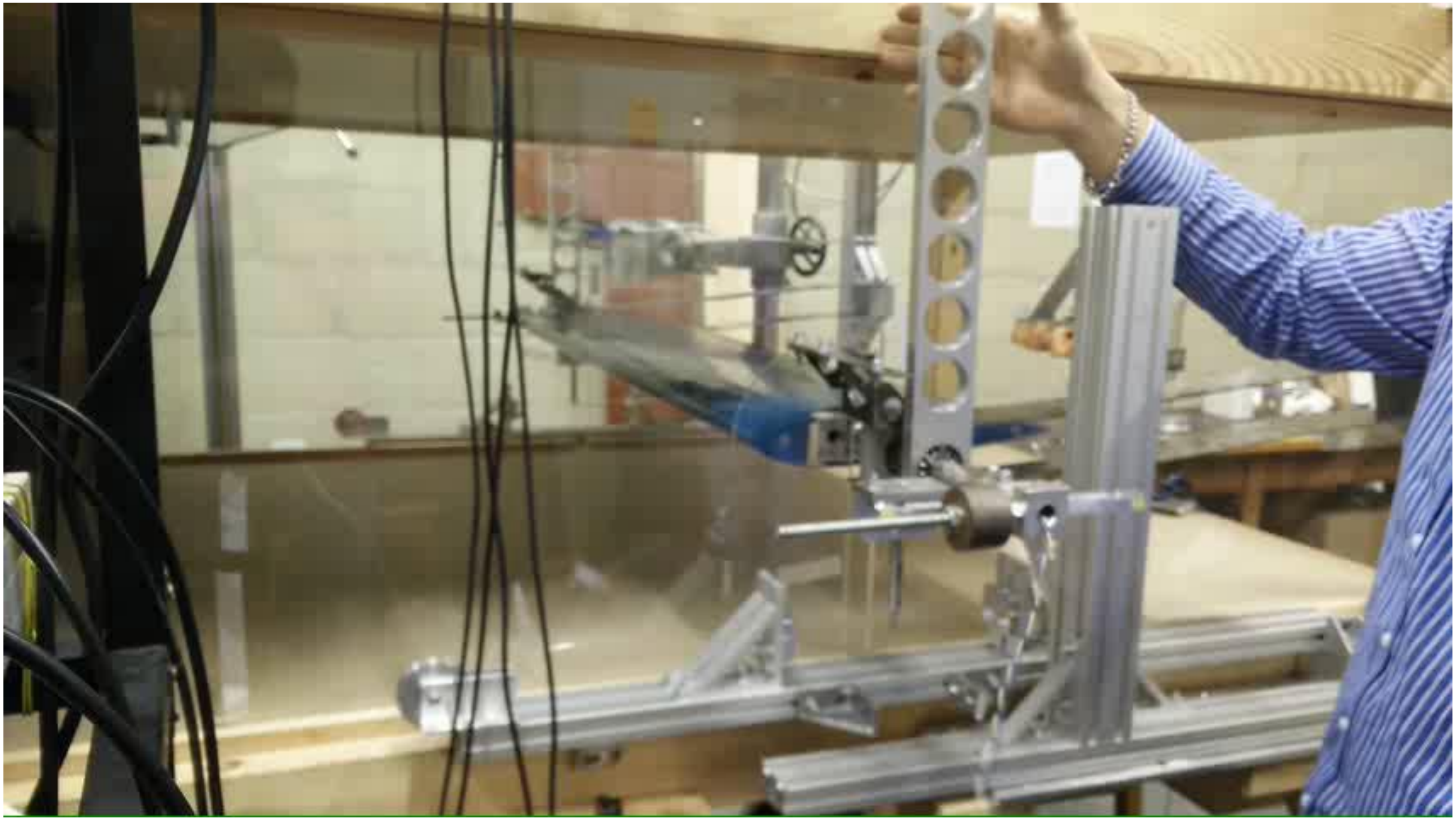
Vibration control – 2 DOF aeroelastic system

Controller OFF - *oscillation*

3

2

1



Controller ON - *oscillation eliminated*

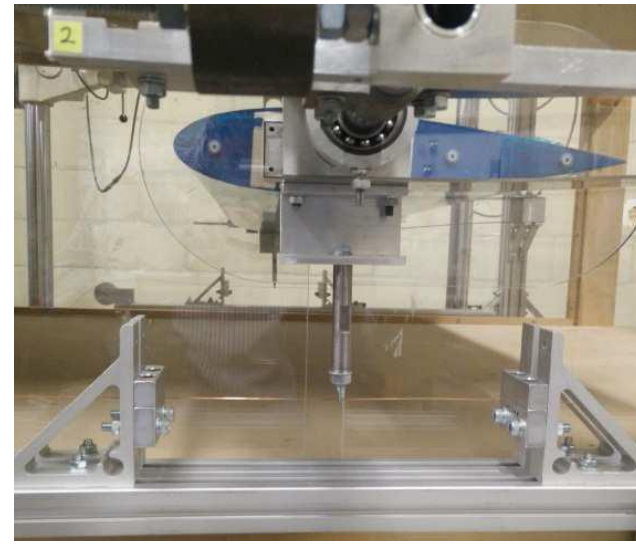
Tensioned-Wire Plunge Nonlinearity Feedback Linearisation

$$\boxed{\dot{\mathbf{x}} = \mathbf{f}\;\;\mathbf{x}\;\; + \mathbf{g}u \qquad y = x_1 \qquad u = \delta}$$

$$x_1' = x_2 \quad x_2' = f_2\;\;\mathbf{x}\;\; + g_2 u \quad \text{Pitch}$$

$$x_3' = x_4 \quad x_4' = f_4\;\;\mathbf{x}\;\; + g_4 u \quad \text{Plunge}$$

$$\left.\begin{array}{ll} x_5' = x_1 - \varepsilon_1 x_5 & x_6' = x_1 - \varepsilon_2 x_6 \\ x_7' = x_3 - \varepsilon_1 x_7 & x_8' = x_3 - \varepsilon_2 x_8 \\ x_9' = u - \varepsilon_1 x_9 & x_{10}' = u - \varepsilon_2 x_{10} \\ x_{11}' = -\varepsilon_3 x_{11} & x_{12}' = -\varepsilon_4 x_{12} \end{array}\right\} \begin{array}{c}\text{Aerodynamic} \\ \text{states}\end{array}$$



States x_5 and x_6 are associated with pitch, x_7 and x_8 are associated with plunge, x_9 and x_{10} are associated with flap motion and x_{11} and x_{12} are associated with gusts.

S. Jiffri, S. Fichera, A. Da-Ronch and J.E. Mottershead, Nonlinear control for the suppression of flutter in a nonlinear aeroelastic system, *AIAA Journal of Guidance, Control and Dynamics*, in preparation.

Pitch Linearisation

Input-output linearisation with pitch displacement chosen as the output y

$$z_1 = y = x_1 \qquad z_2 = z_1' = y' = x_1'$$

$$z_2' = y'' = x_2' = f_2(\mathbf{x}) + g_2 u$$

Relative degree r=2. When r is less than the number of states the system can only be partly linearised.

In matrix form,

$$\begin{Bmatrix} z_1' \\ z_2' \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \end{Bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v, \qquad v = f_2(\mathbf{x}) + g_2 u,$$

Linear and nonlinear terms are located in $f_2(\mathbf{x})$. v denotes the artificial input – it is the term that achieves the desired linear control objective.

Pole Placement

The artificial input may be defined as,

$$v = -k_1 z_1 - k_2 z_2,$$

Choosing $k_1 = \omega_n^2,\ k_2 = 2\zeta_\alpha \omega_n$ will assign the natural frequency ω_n and damping ζ_α. Then,

$$\begin{Bmatrix} z_1' \\ z_2' \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \end{Bmatrix}.$$

And the physical nonlinear input is given by,

$$u = \frac{1}{g_2}\left(v - f_2(\mathbf{x})\right) = \frac{1}{g_2}\left(-k_1 z_1 - k_2 z_2 - f_2(\mathbf{x})\right).$$

This input cancels the system dynamics and implements the linear control requirement.

Internal Dynamics

A linear coordinate transformation is carried out to obtain the system in *Normal Form* – such that the input u does not appear explicitly,

$$\mathbf{z} = \mathbf{Tx}, \qquad \mathbf{T}_{j,j} = 1 \quad j = 1:12,$$

$$\mathbf{T}_{4\,9\,10,:}\,\mathbf{g} = \mathbf{0}$$

remaining terms equal to 0.

The zero dynamics is then obtained by setting the controlled coordinates to zero, i.e. z_1=z_2=0,

$$z_3' = z_4, \quad z_4' = -\frac{g_4}{g_2} f_2 \; \mathbf{z} \; + f_4 \; \mathbf{z} \; , \quad z_5' = -\varepsilon_1 z_5, \quad z_6' = -\varepsilon_2 z_6,$$

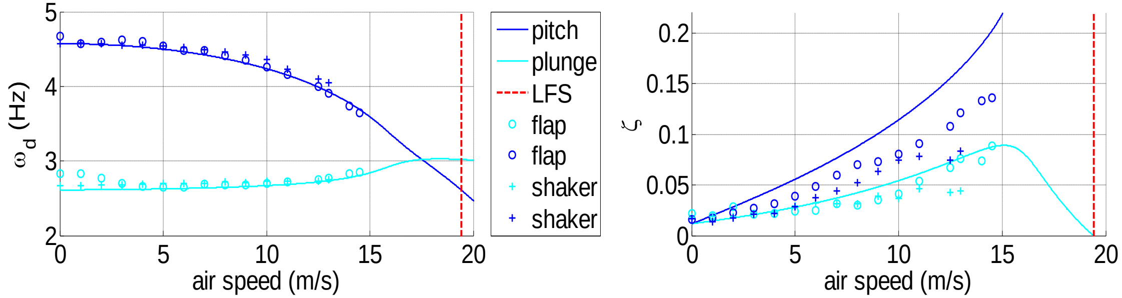
$$z_7' = z_3 - \varepsilon_1 z_7, \quad z_8' = z_3 - \varepsilon_2 z_8, \quad z_9' = -\frac{1}{g_2} f_2 \; \mathbf{z} \; - \varepsilon_1 z_9,$$

$$z_{10}' = -\frac{1}{g_2} f_2 \; \mathbf{z} \; - \varepsilon_2 z_{10}, \quad z_{11}' = -\varepsilon_3 z_{11}, \quad z_{12}' = -\varepsilon_4 z_{12},$$

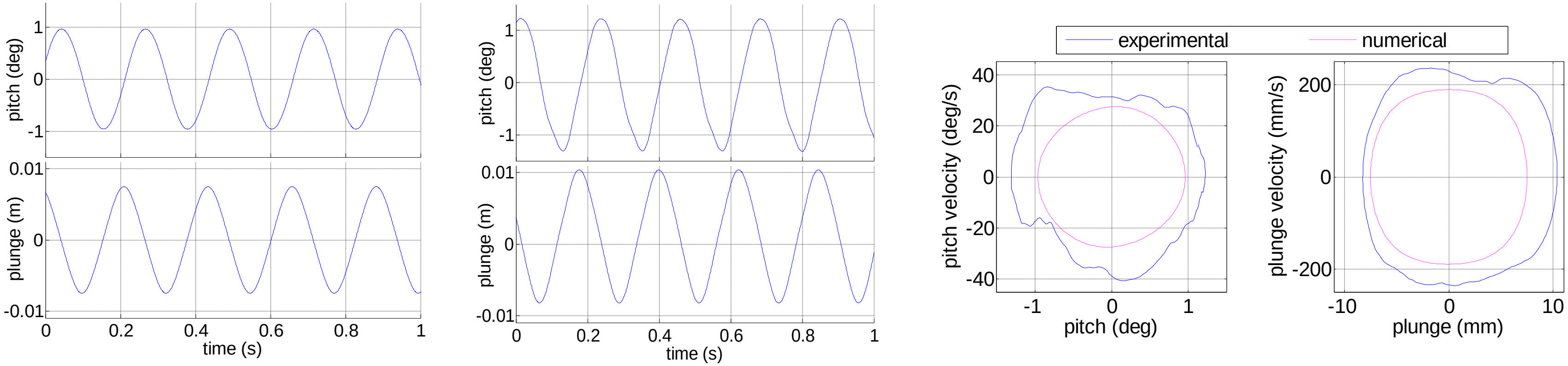
Stability of the zero dynamics must be examined to ensure the stability of the nonlinear controller.

Tuned Numerical Model

Linear frequency-domain tests:



Nonlinear time-domain tests:

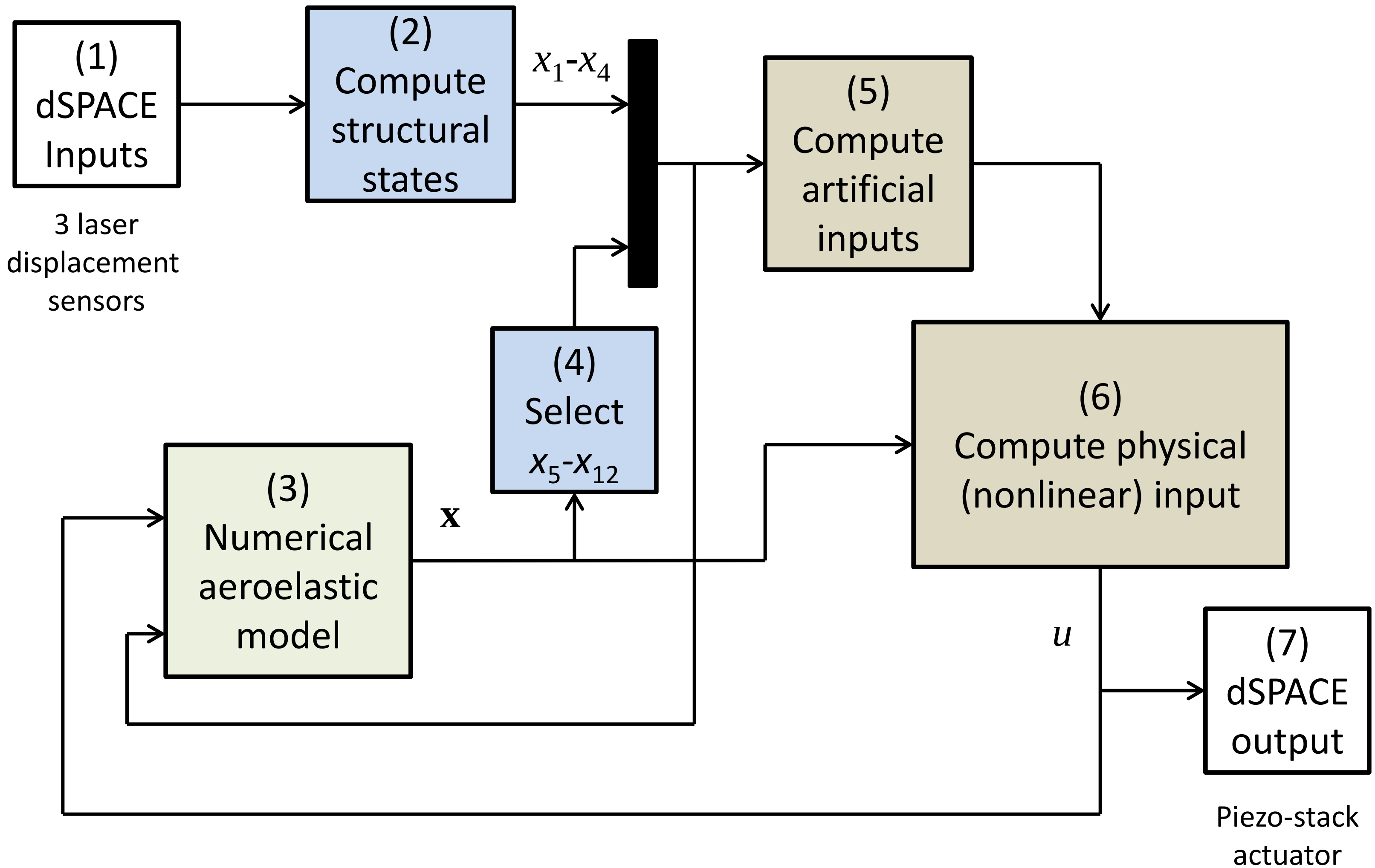


Numerical

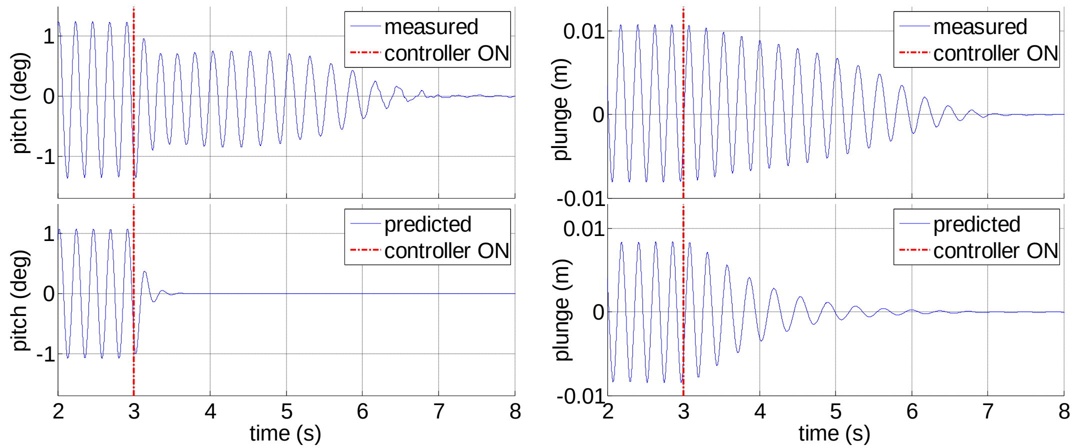
Experimental

Phase portrait

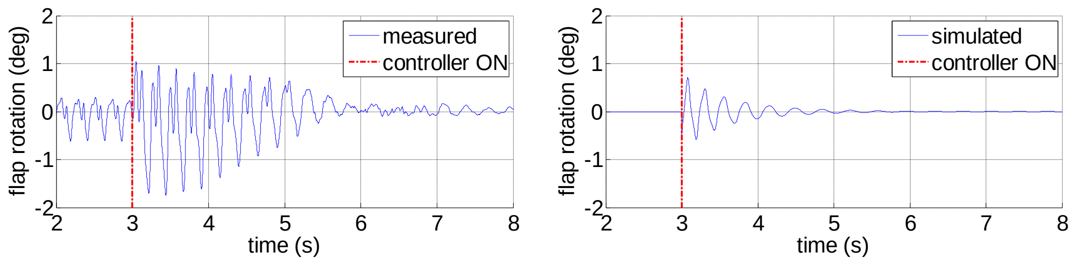
Embedding the Numerical Model in the Aeroelastic Control Loop



Test Results
Assigned Damping at ζ_α=0.3, U=15m/s

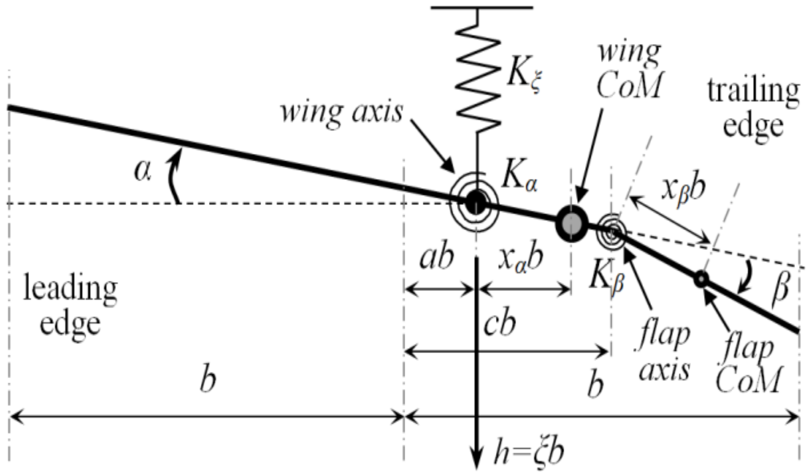


Closed-loop response



Flap motion

Feedback Linearisation
Aerofoil with Non-smooth Pitch Nonlinearity



Aerodynamic model with 8 states, 6 structural (pitch, plunge, flap) and 2 aerodynamic states (Edwards, 1979).

Output y: plunge displacement ξ. Input u: flap command β_{com}.

Edwards, J. W., Ashley, H., and Breakwell, J. V. "Unsteady aerodynamic modeling for arbitrary motions," *AIAA Journal*, Vol. 17, No. 4, 1979, pp. 365-374.

S. Jiffri, P. Paoletti and J.E. Mottershead, Feedback linearization in systems with non-smooth nonlinearities, *AIAA Journal of Guidance, Control and Dynamics*, in press.

Pitch Nonlinearity

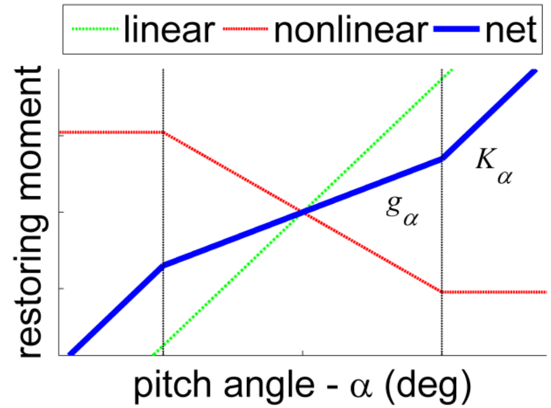
$$f_{nl} = \begin{Bmatrix} -\lambda K_\alpha \alpha, & |\alpha| \le g_\alpha \\ -\lambda K_\alpha g_\alpha, & \alpha > g_\alpha \\ \lambda K_\alpha g_\alpha, & \alpha < -g_\alpha \end{Bmatrix} \lambda \le 1$$

g_α defines the initial (lower) stiffness region on either side of α=0°

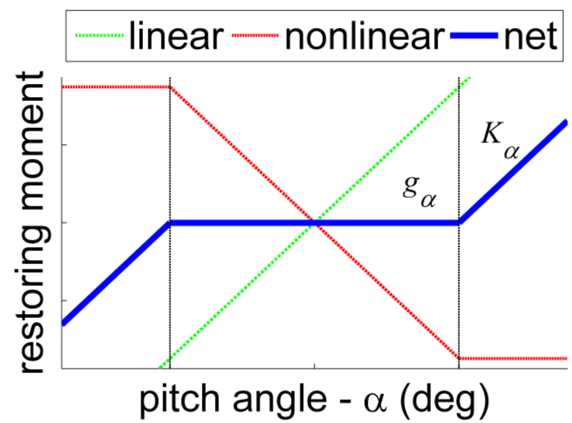
Inner region $|\alpha| \le g_\alpha$: Net stiffness=$(1-\lambda)K_\alpha$
Outer region $|\alpha| > g_\alpha$: Net stiffness = K_α

λ=1 produces freeplay. $\lambda \le 1$ produces piecewise nonlinearity

K_α : chosen as the pitch stiffness of the desired linear system.



Piecewise nonlinearity



Freeplay

Feedback Linearisation
Non-smooth Nonlinear System Parameters

$$\dot{\mathbf{x}} = \underline{\mathbf{f}}(\mathbf{x}) + \underline{\mathbf{g}}(\mathbf{x})u, \quad \text{where } \underline{\mathbf{f}}(\mathbf{x}) = \begin{Bmatrix} \dot{\mathbf{q}} \\ \mathbf{\Psi q} + \mathbf{\Phi}\dot{\mathbf{q}} + \mathbf{\Lambda q}_a + \mathbf{\Omega f}_{nl} \\ \mathbf{E}_1\mathbf{q} + \mathbf{E}_2\dot{\mathbf{q}} + \mathbf{F}_p\mathbf{q}_a \end{Bmatrix}, \quad \underline{\mathbf{g}}(\mathbf{x}) = \begin{bmatrix} \mathbf{0} \\ \mathbf{\Pi} \\ \mathbf{0} \end{bmatrix}$$

$$\mathbf{T} = \left[\begin{array}{ccc|c} \multicolumn{3}{c|}{\boxed{\mathbf{T}_{pl}\ (2\times 2n)}} & \\ \hline \mathbf{0}\ (n-1)\times 1 & \mathbf{I}\ (n-1)\times(n-1) & \mathbf{0}\ (n-1)\times n & \mathbf{0}\ 2n\times 2 \\ \multicolumn{2}{c}{\mathbf{0}\ (n-1)\times n} & \boxed{\ker(\mathbf{\Pi})\ (n-1)\times n} & \\ \hline \multicolumn{3}{c|}{\mathbf{0}\ 2\times 2n} & \mathbf{I}\ 2\times 2 \end{array}\right]$$

Transformation for the linearised part

Input u eliminated in the internal dynamics

No requirement to differentiate the non-smooth nonlinearity.

The transformation matrix $\mathbf{T}$ is invertible.

The usual smoothness requirement on the nonlinearity can be removed.

Stability of the Plunge Zero Dynamics

The zero dynamics are found to take the form:

$$\dot{\underset{\sim}{\mathbf{z}}} = \widehat{\mathbf{A}}\underset{\sim}{\mathbf{z}} + \widehat{\mathbf{b}}\,\psi\left(z_3\right) \qquad \psi\left(z_3\right) = \begin{cases} z_3, & \left|z_3\right| \le g_\alpha \\ g_\alpha, & z_3 > g_\alpha \\ -g_\alpha, & z_3 < -g_\alpha \end{cases}$$

$$\underset{\sim}{\mathbf{z}} = \mathbf{z}\left(3{:}8\right)$$

The equilibrium points are found when:

$$\begin{cases} \left(\widehat{\mathbf{A}} + \widehat{\mathbf{b}}\mathbf{e}_1^T\right)\underset{\sim}{\mathbf{z}}_{eq} = \mathbf{0}; \quad \mathbf{e}_1^T\underset{\sim}{\mathbf{z}} = z_3 \\ \widehat{\mathbf{A}}\underset{\sim}{\mathbf{z}}_{eq} + \widehat{\mathbf{b}}g_a = \mathbf{0} \\ \widehat{\mathbf{A}}\underset{\sim}{\mathbf{z}}_{eq} - \widehat{\mathbf{b}}g_a = \mathbf{0} \end{cases}$$

Assuming non-singular $\widehat{\mathbf{A}}$ and $\widehat{\mathbf{A}} + \widehat{\mathbf{b}}\mathbf{e}_1^T$ and solving above for each of the 3 conditions,

$$\underset{\sim}{\mathbf{z}}_{eq} = \begin{cases} \mathbf{0}, & \left|z_3\right| \le g_\alpha \\ -\widehat{\mathbf{A}}^{-1}\widehat{\mathbf{b}}g_\alpha, & z_3 > g_\alpha \\ \widehat{\mathbf{A}}^{-1}\widehat{\mathbf{b}}g_\alpha, & z_3 < -g_\alpha \end{cases}$$

Stability of the Plunge Zero Dynamics

There are 2 non-zero equilibria.

Lyapunov function based around the particular equilibrium point:

$$\hat{\underset{\sim}{\mathbf{z}}} = \underset{\sim}{\mathbf{z}} - \underset{\sim}{\mathbf{z}}_{eq}, \qquad V = \tfrac{1}{2}\hat{\underset{\sim}{\mathbf{z}}}^T \mathbf{P} \hat{\underset{\sim}{\mathbf{z}}}, \qquad \mathbf{P} > 0, \qquad \mathbf{P} = \mathbf{P}^T$$

The derivative of V for each of the three regions may be expressed as (Shevitz and Paden, 1994):

$$\dot{\tilde{V}} = \begin{cases} \hat{\underset{\sim}{\mathbf{z}}}^T \mathbf{P} \;\; \widehat{\mathbf{A}} + \widehat{\mathbf{b}}\mathbf{e}_1^T \;\; \hat{\underset{\sim}{\mathbf{z}}} + \underset{\sim}{\mathbf{z}}_{eq} \;\; , & |z_3| \le g_\alpha \\ \hat{\underset{\sim}{\mathbf{z}}}^T \mathbf{P} \;\; \widehat{\mathbf{A}}\hat{\underset{\sim}{\mathbf{z}}} + \widehat{\mathbf{A}}\underset{\sim}{\mathbf{z}}_{eq} + \widehat{\mathbf{b}} g_\alpha \;\; , & z_3 > g_\alpha \\ \hat{\underset{\sim}{\mathbf{z}}}^T \mathbf{P} \;\; \widehat{\mathbf{A}}\hat{\underset{\sim}{\mathbf{z}}} + \widehat{\mathbf{A}}\underset{\sim}{\mathbf{z}}_{eq} - \widehat{\mathbf{b}} g_\alpha \;\; , & z_3 < -g_\alpha \end{cases} \quad \text{for} \quad \begin{cases} \underset{\sim}{\mathbf{z}}_{eq} = 0 \\ \underset{\sim}{\mathbf{z}}_{eq} = -\widehat{\mathbf{A}}^{-1}\widehat{\mathbf{b}} g_\alpha \\ \underset{\sim}{\mathbf{z}}_{eq} = \widehat{\mathbf{A}}^{-1}\widehat{\mathbf{b}} g_\alpha \end{cases}$$

These derivatives must be strictly negative for asymptotic stability. 3 z_3 stability regions, for each of the 3 equilibria → 9 equations to be investigated.

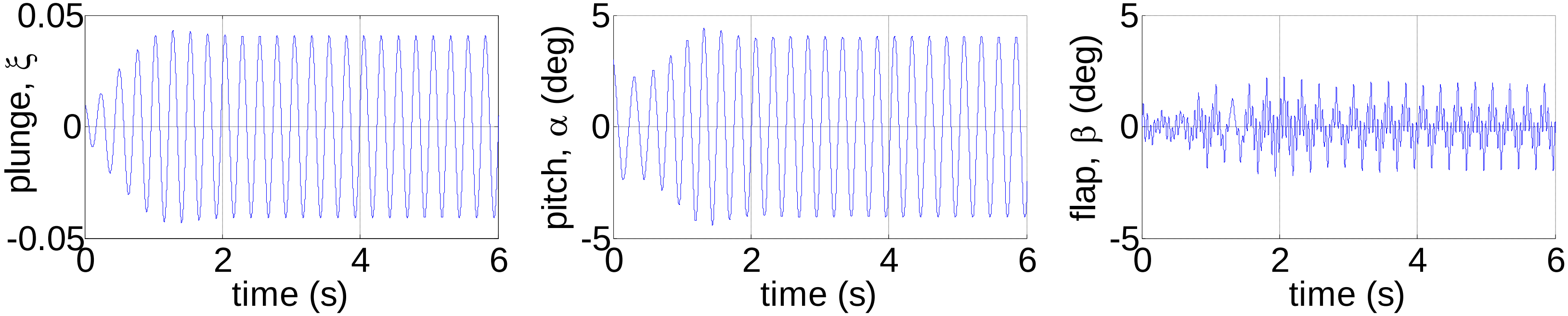
Shevitz, D., and Paden, B. "Lyapunov stability theory of nonsmooth systems," *IEEE Transactions on Automatic Control*, Vol. 39, No. 9, 1994, pp. 1910-1914.

S. Jiffri, P. Paoletti and J.E. Mottershead, Feedback linearization in systems with non-smooth nonlinearities, *AIAA Journal of Guidance, Control and Dynamics*, in press.

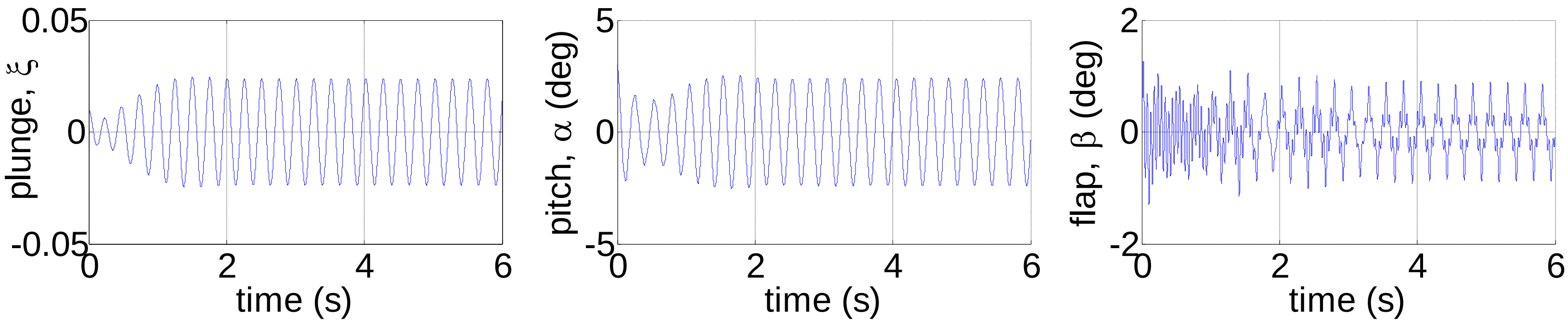
Uncontrolled Nonlinear Response

Reduced air speed U*=2.0

Plunge Pitch Flap



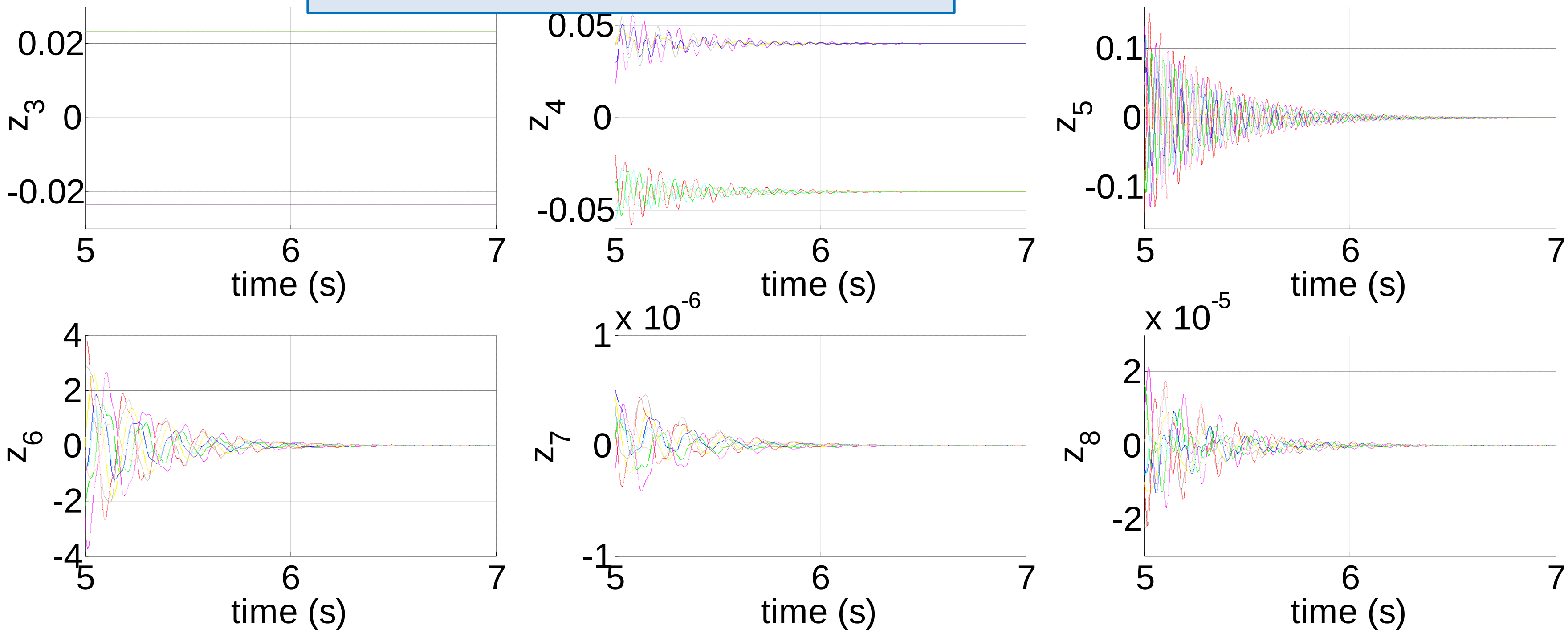
(a) Freeplay



(b) Piecewise Linear Stiffness

Zero-dynamics: Freeplay

The trivial equilibrium point , $\underset{\sim}{\mathbf{z}}_{eq} = 0$, is unstable. The non-trivial equilibria $\underset{\sim}{\mathbf{z}}_{eq} = \pm\widehat{\mathbf{A}}^{-1}\widehat{\mathbf{b}}g_\alpha$ are stable in each of the 3 regions.

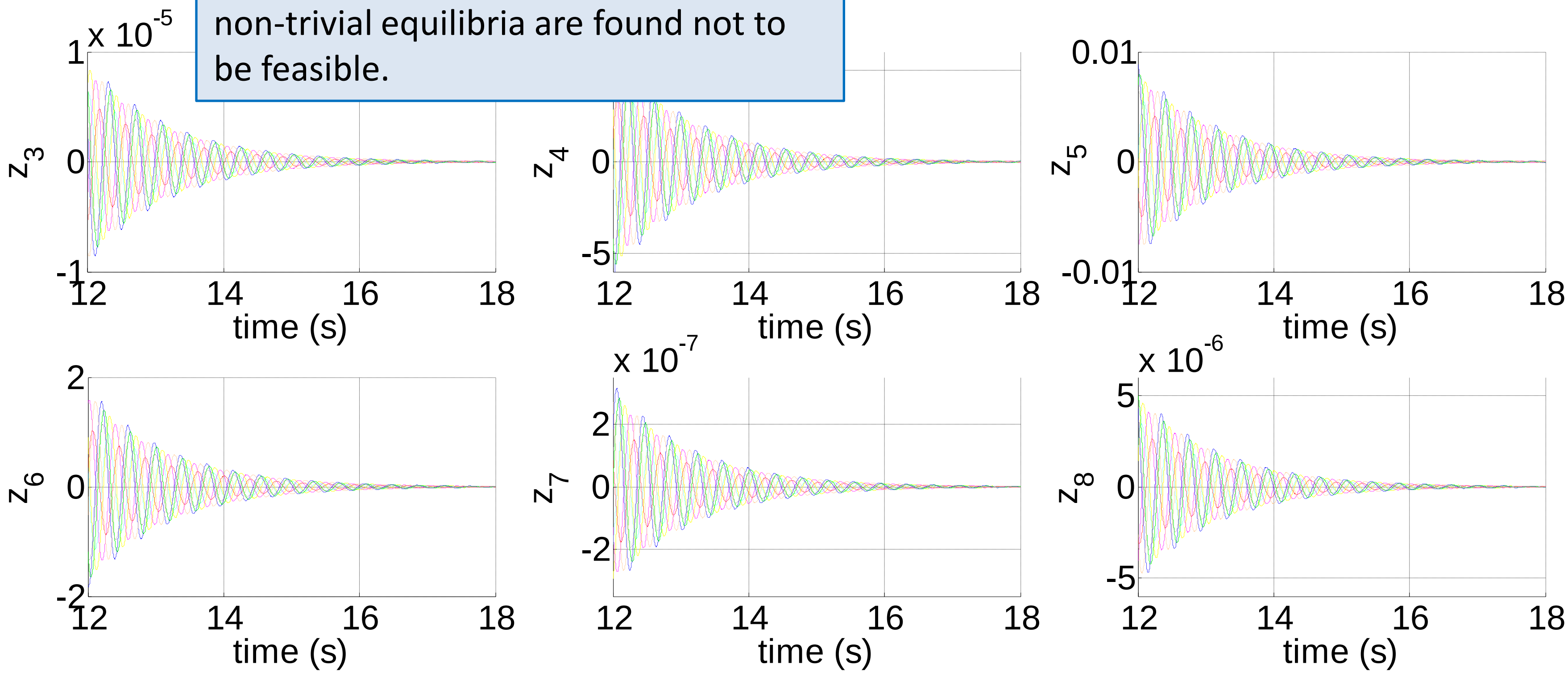


Reduced air speed $U^*=2.0$

Superimposed simulated time-domain responses with randomly generated initial conditions

Zero-dynamics: Piecewise Linear Stiffness

The trivial equilibrium point, $\mathbf{z}_{eq}=\mathbf{0}$, is stable in each of the 3 regions. The non-trivial equilibria are found not to be feasible.



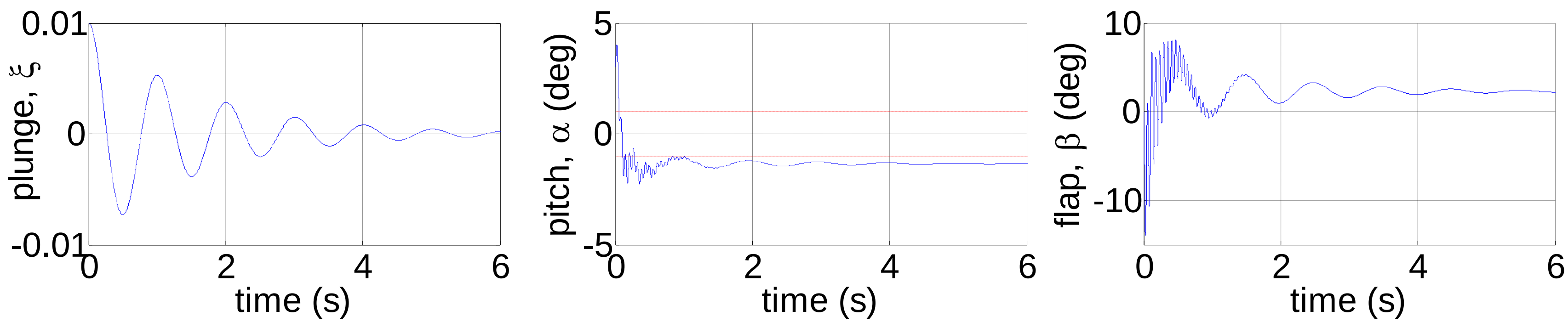
Reduced air speed $U^{*}=2.0$

Superimposed simulated time-domain responses with randomly generated initial conditions

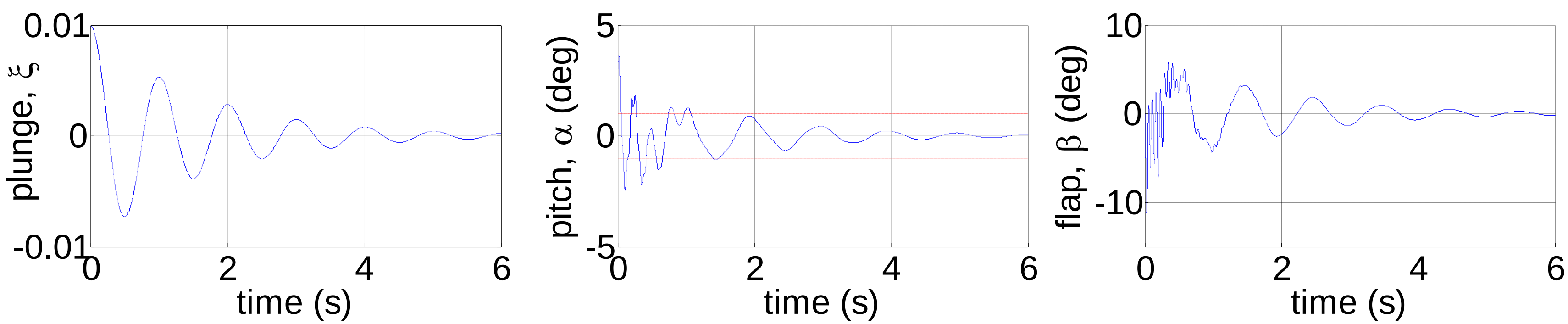
Time-domain Closed-loop Response with Feedback Linearisation

Reduced air speed $U^{*}=2.0$

Assigned frequency and damping $\omega_{n_\xi} = 1\mathrm{Hz},\ \zeta_\xi = 0.1$



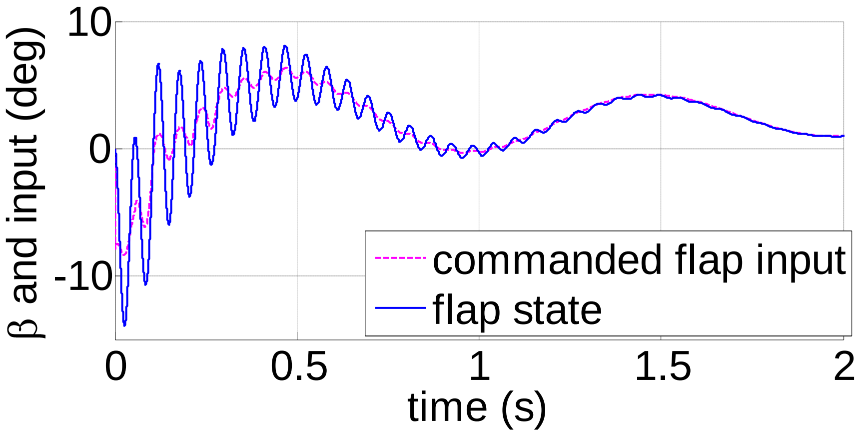
(a) Freeplay



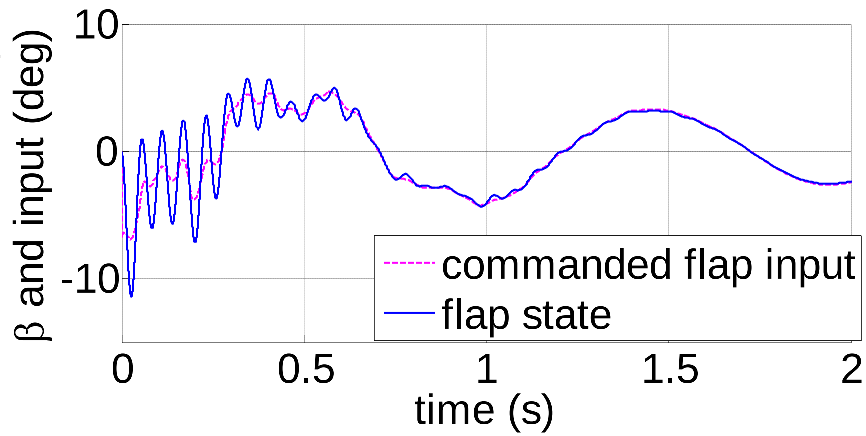
(b) Piecewise Linear Stiffness

Commanded and Actual Flap Angles

Reduced air speed $U^{*}=2.0$

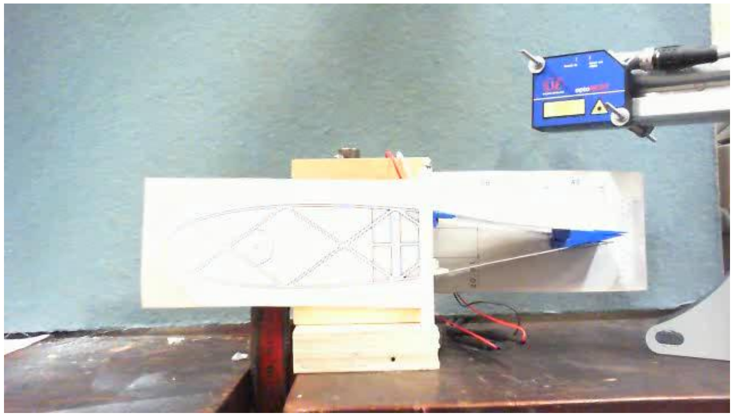


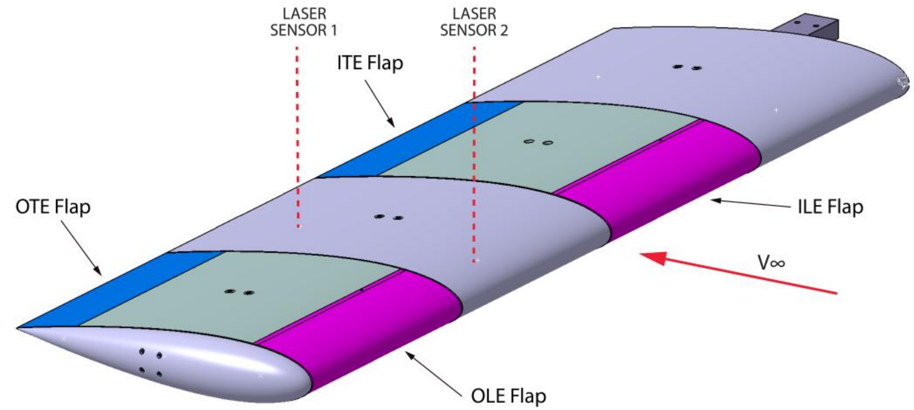
(a) Freeplay

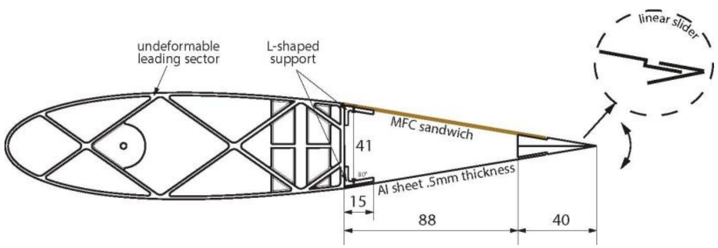


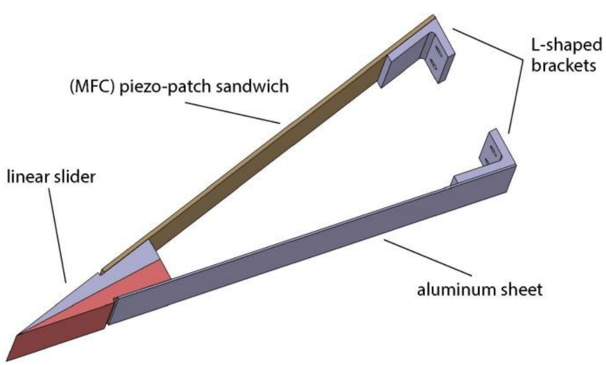
(b) Piecewise Linear Stiffness

High Bandwidth Morphing Actuator (HBMA) & Modular Flexible Aeroelastic Wing (ModFlex)









Conclusions

- Linear flutter suppression demonstrated by active control using the method of receptances – based on data from a vibration test with no requirement to evaluate or to know the system matrices.
- Nonlinear aeroelastic system - plunge nonlinearity representative of future aircraft with very flexible wings.
- Feedback linearisation - demonstration of LCO suppression.
- Non-smooth nonlinearities – piecewise stiffness and freeplay.
- Feedback linearisation for non-smooth nonlinearity.
- Multiple equilibria – zero and nonzero.
- Numerical demonstration.
- MODFLEX wing in preparation – multiple control surfaces, conventional motor-driven flaps, morphing using piezo-benders .

Selected Papers

- Y.M. Ram and J.E. Mottershead, The receptance method in active vibration control, *American Institute of Aeronautics and Astronautics Journal*, 45(3), 2007, 562-567.
- M. Ghandchi Tehrani, R.N.R. Elliott and J.E. Mottershead, Partial pole placement in structures by the method of receptances: theory and experiments, *Journal of Sound and Vibration*, 329(24), 2010, 5017-5035.
- M. Ghandchi Tehrani, J.E. Mottershead, A.T. Shenton and Y.M. Ram, Robust pole placement in structures by the method of receptances, *Mechanical Systems and Signal Processing*, 25(1), 2011, 112–122.
- Y.M. Ram and J.E. Mottershead, Multiple-input active vibration control by partial pole placement using the method of receptances, *Mechanical Systems and Signal Processing*, 40, 2013, 727-735.
- S. Jiffri, P. Paoletti, J.E. Cooper and J.E. Mottershead, Feedback linearisation for nonlinear vibration problems, *Shock and Vibration*, vol. 2014, Article ID 106531, 16 pages, 2014. doi:10.1155/2014/106531.
- X. Wei and J.E. Mottershead, Block-decoupling vibration control using eigenstructure assignment, *Mechanical Systems and Signal Processing*, http://dx.doi.org/10.1016/j.ymssp.2015.03.028i .
- X. Wei, J.E. Mottershead and Y.M. Ram, Partial pole placement by feedback control with inaccessible degrees of freedom, *Mechanical Systems and Signal Processing*, in press.
- S. Jiffri, P. Paoletti and J.E. Mottershead, Feedback linearization in systems with non-smooth nonlinearities, *AIAA Journal of Guidance, Control and Dynamics*, in press.
- S. Jiffri, S. Fichera, A. Da-Ronch and J.E. Mottershead, Nonlinear control for the suppression of flutter in a nonlinear aeroelastic system, *AIAA Journal of Guidance, Control and Dynamics*, in preparation.