Pole Placement and Active Vibration Control in Aeroelasticity

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Receptance Method

- Dynamic stiffnesses → Receptances:

\[ s^2 M + sC + K \quad x \quad s = f \quad s \]

\[ H(s) = (s^2 M + sC + K)^{-1} \]

- No need to evaluate or to know the system matrices \( M, C, K \).
- Any input-output transfer function may be used – dynamics of actuators, sensors, filters etc. included in the measurement.
Receptance Method
Partial Pole Placement Problem

Open–loop and closed-loop systems:

\[
\begin{align*}
\lambda_k^2M + \lambda_k C + K \ v_k &= 0 \\
\mu_k^2M + \mu_k C + K \ w_k &= Bu(t) \\
u(t) &= \mu_k F^T + G^T \ w_k
\end{align*}
\]

\[
k = 1, 2, \ldots, 2n
\]

Assigned eigenvalues \( \mu_k \) \( p \) are distinct from eigenvalues \( \lambda_k \) \( 2n \).

While eigenvalues \( \mu_k = \lambda_k \) \( k = p + 1, p + 2, \ldots, 2n \) are unchanged.
Unchanged Eigenvalues

\[
\lambda_k^2 M + \lambda_k C + K \ w_k = B \ \lambda_k F^T + G^T \ w_k
\]

\[
B = \begin{bmatrix} b_1 & b_2 & \cdots & b_m \end{bmatrix} \quad F = \begin{bmatrix} f_1 & f_2 & \cdots & f_m \end{bmatrix} \quad G = \begin{bmatrix} g_1 & g_2 & \cdots & g_m \end{bmatrix}
\]

May be re-written as,

\[
\lambda_k^2 M + \mu_k C + K \ \tilde{w}_k = \phi_1 \lambda_k f_1^T + g_1^T + b_2 \lambda_k f_2^T + g_2^T + \cdots + b_m \lambda_k f_m^T + g_m^T \ \tilde{w}_k
\]

Non-trivial solution: \( w_k = v_k \)

\[
\phi_1 \lambda_k f_1^T + g_1^T + \cdots + b_m \lambda_k f_m^T + g_m^T \ v_k = 0 \quad k = p + 1, p + 2, \ldots, 2n
\]

\[
\begin{bmatrix}
\lambda_k v_k^T & 0 & \cdots & 0 & v_k^T & 0 & \cdots & 0 \\
0 & \lambda_k v_k^T & \cdots & 0 & 0 & v_k^T & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & \lambda_k v_k^T & 0 & 0 & \cdots & v_k^T
\end{bmatrix}
\begin{bmatrix}
f_1 \\
\vdots \\
f_m \\
g_1 \\
\vdots \\
g_m
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
\vdots \\
0
\end{bmatrix}
\]
Assigned Eigenvalues

\[ w_k = H \xi_k \bigg( \mu_k f_1^T + g_1^T \bigg) + b_2 \xi_2 f_2^T + g_2^T + \ldots + b_m \xi_m f_m^T + g_m^T \bigg) w_k \quad k = 1, 2, \ldots, p \]

Let \[ r_{\mu_k, j} = H \xi_k b_j \] and \[ \alpha_{\mu_k, j} = \xi_k f_j^T + g_j^T w_k \quad j = 1, 2, \ldots, m \]

Then

\[
\begin{bmatrix}
\mu_k w_k^T & 0 & \ldots & 0 & w_k^T & 0 & \ldots & 0 \\
0 & \mu_k w_k^T & \ldots & 0 & 0 & w_k^T & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & \mu_k w_k^T & 0 & 0 & \ldots & w_k^T \\
\end{bmatrix}
\begin{bmatrix}
f_1 \\ \vdots \\ f_m \\ g_1 \\ \vdots \\ g_m \\
\end{bmatrix}
= 
\begin{bmatrix}
\alpha_{\mu_k, 1} \\
\alpha_{\mu_k, 2} \\
\vdots \\
\alpha_{\mu_k, m} \\
\end{bmatrix}
\]

where \[ w_k = \alpha_{\mu_k, 1} r_{\mu_k, 1} + \alpha_{\mu_k, 1} r_{\mu_k, 2} + \ldots + \alpha_{\mu_k, 1} r_{\mu_k, m} \]

The closed-loop mode shape is determined by the choice of \[ \alpha_{\mu_k, j} \]
The control gains are then given by the solution of,

\[
P_k = \begin{bmatrix} \mu_k w_k^T & 0 & \cdots & 0 & w_k^T & 0 & \cdots & 0 \\ 0 & \mu_k w_k^T & \cdots & 0 & 0 & w_k^T & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mu_k w_k^T & 0 & 0 & \cdots & w_k^T \end{bmatrix}
\]

\[
Q_k = \begin{bmatrix} \lambda_k v_k^T & 0 & \cdots & 0 & v_k^T & 0 & \cdots & 0 \\ 0 & \lambda_k v_k^T & \cdots & 0 & 0 & v_k^T & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_k v_k^T & 0 & 0 & \cdots & v_k^T \end{bmatrix}
\]

General Procedure

- Measure the open loop input-output FRF over a desired frequency range.
- Fit MIMO rational fraction polynomials to the measure FRF and obtain the input-output transfer functions.
- Select force distribution vectors $b_k(s)$.
- Apply the Receptance Method to obtain unknown gains, $g_k$, $f_k$.

\[
\begin{bmatrix}
P_1 \\ 
\vdots \\ 
P_p \\ 
Q_{p+1} \\ 
\vdots \\ 
Q_{2n}
\end{bmatrix}
\begin{bmatrix}
f_1 \\ 
\vdots \\ 
f_m \\ 
g_1 \\ 
\vdots \\ 
g_m
\end{bmatrix}
= 
\begin{bmatrix}
\alpha_1 \\ 
\vdots \\ 
\alpha_p \\ 
0 \\ 
\vdots \\ 
0
\end{bmatrix}
\]

- Implementation of the controller using dSPACE in real time.
Flutter Suppression
Wind-Tunnel Aerofoil Rig

Aerofoil

Torsion Bar

Vertical Stiffness

Torsional Stiffness

Flap

V-stack piezo actuator
Open loop FRFs include not only the dynamics of the aerofoil system but also the power amplifier, the actuator, the sensors and the effects of A/D and D/A conversion, numerical differentiation (Simulink/dSPACE) of displacements and high- and low-pass Butterworth filters with cut-off frequencies of 1Hz and 35 Hz.
Poles assigned: \( \mu_{pitch} = -1.5 \pm 38i, \quad \mu_{heave} = -0.7 \pm 23i \)

at wind speed, 7m/s.
Flutter Margin

\[ F = \left( \frac{\tilde{\omega}^2_2 - \tilde{\omega}^2_1}{2} \right) + \left( \frac{\sigma^2_2 - \sigma^2_1}{2} \right) \right]^2 + 4\sigma_1 \sigma_2 \left[ \left( \frac{\tilde{\omega}^2_2 + \tilde{\omega}^2_1}{2} \right) + 2 \left( \frac{\sigma^2_2 + \sigma^2_1}{2} \right)^2 \right] \]

\[ \tilde{\omega}_j = \omega_j \sqrt{1 - \zeta^2_j} \]

\[ \sigma_j = -\zeta_j \omega_j \quad j = 1,2 \]

Zimmerman N.H. and Weissenburger J.T. (1964)

Quadratic flutter speed prediction.

Predicted flutter speed increased from 17 m/s to 20 m/s.

Separation of pitch and heave frequencies.

\[ \mathbf{g} = \begin{bmatrix} -28.2 \\ 29.5 \end{bmatrix} \quad \mathbf{f} = \begin{bmatrix} 0.0284 \\ 0.0232 \end{bmatrix} \]
Vibration control – 2 DOF aeroelastic system

Controller OFF - oscillation

Controller ON - oscillation eliminated
Tensioned-Wire Plunge Nonlinearity Feedback Linearisation

\[ \dot{x} = f \ x + gu \]

\[ y = x_1 \quad u = \delta \]

\[ x'_1 = x_2 \quad x'_2 = f_2 \ x + g_2 u \quad \text{Pitch} \]

\[ x'_3 = x_4 \quad x'_4 = f_4 \ x + g_4 u \quad \text{Plunge} \]

\[ x'_5 = x_1 - \varepsilon_1 x_5 \quad x'_6 = x_1 - \varepsilon_2 x_6 \]

\[ x'_7 = x_3 - \varepsilon_1 x_7 \quad x'_8 = x_3 - \varepsilon_2 x_8 \quad \text{Aerodynamic states} \]

\[ x'_9 = u - \varepsilon_1 x_9 \quad x'_{10} = u - \varepsilon_2 x_{10} \]

\[ x'_{11} = -\varepsilon_3 x_{11} \quad x'_{12} = -\varepsilon_4 x_{12} \]

States \( x_5 \) and \( x_6 \) are associated with pitch, \( x_7 \) and \( x_8 \) are associated with plunge, \( x_9 \) and \( x_{10} \) are associated with flap motion and \( x_{11} \) and \( x_{12} \) are associated with gusts.

Pitch Linearisation

Input-output linearisation with pitch displacement chosen as the output $y$

$$z_1 = y = x_1, \quad z_2 = z'_1 = y' = x'_1$$

$$z'_2 = y'' = x'_2 = f_2 \ x + g_2 u$$

Relative degree $r=2$. When $r$ is less than the number of states the system can only be partly linearised.

In matrix form,

$$\begin{bmatrix} z'_1 \\ z'_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v, \quad \begin{bmatrix} v \\ 0 \end{bmatrix} = f_2 \ x + g_2 u,$$

Linear and nonlinear terms are located in $f_2(x)$. $v$ denotes the artificial input – it is the term that achieves the desired linear control objective.
The artificial input may be defined as,

\[ v = -k_1 z_1 - k_2 z_2, \]

Choosing \( k_1 = \omega_n^2, \quad k_2 = 2\zeta_\alpha \omega_n \) will assign the natural frequency \( \omega_n \) and damping \( \zeta_\alpha \). Then,

\[
\begin{pmatrix}
    z_1' \\
    z_2'
\end{pmatrix} =
\begin{bmatrix}
    0 & 1 \\
    -k_1 & -k_2
\end{bmatrix}
\begin{pmatrix}
    z_1 \\
    z_2
\end{pmatrix}.
\]

And the physical nonlinear input is given by,

\[
u = \frac{1}{g_2} \left( v - f_2 x \right) = \frac{1}{g_2} \left( -k_1 z_1 - k_2 z_2 - f_2 x \right).
\]

This input cancels the system dynamics and implements the linear control requirement.
Internal Dynamics

A linear coordinate transformation is carried out to obtain the system in Normal Form – such that the input $u$ does not appear explicitly,

$$z = Tx, \quad T_{j,j} = 1 \quad j = 1:12,$$

$$T_{4910} \cdot g = 0$$

remaining terms equal to 0.

The zero dynamics is then obtained by setting the controlled coordinates to zero, i.e. $z_1 = z_2 = 0$,

$$z'_3 = z_4, \quad z'_4 = -\frac{g_4}{g_2} f_2 \, z + f_4 \, z, \quad z'_5 = -\varepsilon_1 z_5, \quad z'_6 = -\varepsilon_2 z_6,$$

$$z'_7 = z_3 - \varepsilon_1 z_7, \quad z'_8 = z_3 - \varepsilon_2 z_8, \quad z'_9 = -\frac{1}{g_2} f_2 \, z - \varepsilon_1 z_9,$$

$$z'_{10} = -\frac{1}{g_2} f_2 \, z - \varepsilon_2 z_{10}, \quad z'_{11} = -\varepsilon_3 z_{11}, \quad z'_{12} = -\varepsilon_4 z_{12},$$

Stability of the zero dynamics must be examined to ensure the stability of the nonlinear controller.
Tuned Numerical Model

**Linear frequency-domain tests:**

- Pitch vs. air speed
- Plunge vs. air speed

**Nonlinear time-domain tests:**

- Pitch vs. time
- Plunge vs. time

**Numerical vs. Experimental**

- Phase portrait comparison
Embedding the Numerical Model in the Aeroelastic Control Loop

1. dSPACE Inputs
2. Compute structural states ($x_1$-$x_4$)
3. Numerical aeroelastic model
4. Select ($x_5$-$x_{12}$)
5. Compute artificial inputs
6. Compute physical (nonlinear) input ($x$)
7. dSPACE output

Additional notes:
- 3 laser displacement sensors
- Piezo-stack actuator
Test Results
Assigned Damping at $\zeta_\alpha=0.3$, $U=15\text{m/s}$

Closed-loop response

Flap motion
Aerodynamic model with 8 states, 6 structural (pitch, plunge, flap) and 2 aerodynamic states (Edwards, 1979).

Output $y$: plunge displacement $\xi$. Input $u$: flap command $\beta_{com}$.


Pitch Nonlinearity

\[ f_{nl} = \begin{cases} 
-\lambda K_\alpha \alpha, & |\alpha| \leq g_\alpha \\
-\lambda K_\alpha g_\alpha, & \alpha > g_\alpha \\
\lambda K_\alpha g_\alpha, & \alpha < -g_\alpha 
\end{cases} \quad \lambda \leq 1 \]

\( g_\alpha \) defines the initial (lower) stiffness region on either side of \( \alpha=0^\circ \)

**Inner region** \( |\alpha| \leq g_\alpha \): Net stiffness = \((1-\lambda)K_\alpha\)

**Outer region** \( |\alpha| > g_\alpha \): Net stiffness = \( K_\alpha \)

\( \lambda = 1 \) produces freeplay. \( \lambda \leq 1 \) produces piecewise nonlinearity

\( K_\alpha \): chosen as the pitch stiffness of the desired linear system.
Feedback Linearisation
Non-smooth Nonlinear System Parameters

\[ \dot{x} = f(x) + g(x)u, \quad \text{where}\; f(x) = \begin{cases} \dot{q} \\ \Psi q + \Phi \dot{q} + \Lambda q_a + \Omega f_{nl} \\ E_1 q + E_2 \dot{q} + F_p q_a \end{cases}, \quad g(x) = \begin{bmatrix} 0 \\ \Pi \end{bmatrix} \]

Transformation for the linearised part

\[
T = T_{pl} = \begin{bmatrix} T_{pl} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & I_{n-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I_{2n} \end{bmatrix}
\]

No requirement to differentiate the non-smooth nonlinearity.

The transformation matrix \( T \) is invertible.

The usual smoothness requirement on the nonlinearity can be removed.

Input \( u \) eliminated in the internal dynamics.
Stability of the Plunge Zero Dynamics

The zero dynamics are found to take the form:

\[ \dot{z} = \hat{A}z + \hat{b}\psi \quad z_3 \]
\[ \hat{z} = \hat{z}_{3:8} \]

\[ \psi \quad z_3 = \begin{cases} z_3, & |z_3| \leq g_\alpha \\ g_\alpha, & z_3 > g_\alpha \\ -g_\alpha, & z_3 < -g_\alpha \end{cases} \]

The equilibrium points are found when:

\[ \hat{A} + \hat{b}e_1^T \quad z_{eq} = 0; \quad e_1^T z = z_3 \]
\[ \hat{A}z_{eq} + \hat{b}g_\alpha = 0 \]
\[ \hat{A}z_{eq} - \hat{b}g_\alpha = 0 \]

Assuming non-singular \( \hat{A} \) and \( \hat{A} + \hat{b}e_1^T \) and solving above for each of the 3 conditions,

\[ z_{eq} = \begin{cases} 0, & |z_3| \leq g_\alpha \\ -\hat{A}^{-1}\hat{b}g_\alpha, & z_3 > g_\alpha \\ \hat{A}^{-1}\hat{b}g_\alpha, & z_3 < -g_\alpha \end{cases} \]
Stability of the Plunge Zero Dynamics

There are 2 non-zero equilibria. Lyapunov function based around the particular equilibrium point:

\[ \hat{z} = z - z_{eq}, \quad V = \frac{1}{2} \hat{z}^T P \hat{z}, \quad P > 0, \quad P = P^T \]

The derivative of \( V \) for each of the three regions may be expressed as (Shevitz and Paden, 1994):

\[
\dot{V} = \begin{cases} 
\hat{z}^T P \hat{A} \hat{z} + \hat{A} z_{eq} + \hat{b} g_{\alpha}, & z_3 > g_{\alpha} \\
\hat{z}^T P \hat{A} \hat{z} + \hat{A} z_{eq} - \hat{b} g_{\alpha}, & z_3 < -g_{\alpha} \\
\hat{z}^T P \hat{A} \hat{z} + \hat{A} z_{eq}, & |z_3| \leq g_{\alpha}
\end{cases}
\]

These derivatives must be strictly negative for asymptotic stability. 3 \( z_3 \) stability regions, for each of the 3 equilibria \( \rightarrow 9 \) equations to be investigated.


Uncontrolled Nonlinear Response

Reduced air speed $U^*=2.0$

(a) Freeplay

(b) Piecewise Linear Stiffness
Zero-dynamics: Freeplay

The trivial equilibrium point, $\mathbf{z}_{eq} = 0$, is unstable. The non-trivial equilibria $\mathbf{z}_{eq} = \pm \hat{\mathbf{A}}^{-1} \hat{\mathbf{b}} g_\alpha$ are stable in each of the 3 regions.

Superimposed simulated time-domain responses with randomly generated initial conditions
The trivial equilibrium point, $z_{eq}=0$, is stable in each of the 3 regions. The non-trivial equilibria are found not to be feasible.

Reduced air speed $U^*=2.0$

Superimposed simulated time-domain responses with randomly generated initial conditions
Time-domain Closed-loop Response with Feedback Linearisation

Reduced air speed $U^*=2.0$

Assigned frequency and damping $\omega_n = 1\text{Hz}, \zeta = 0.1$

(a) Freeplay

(b) Piecewise Linear Stiffness
Commanded and Actual Flap Angles

Reduced air speed $U^* = 2.0$

(a) Freeplay

(b) Piecewise Linear Stiffness
High Bandwidth Morphing Actuator (HBMA) & Modular Flexible Aeroelastic Wing (ModFlex)
Conclusions

• Linear flutter suppression demonstrated by active control using the method of receptances – based on data from a vibration test with no requirement to evaluate or to know the system matrices.

• Nonlinear aeroelastic system - plunge nonlinearity representative of future aircraft with very flexible wings.

• Feedback linearisation - demonstration of LCO suppression.

• Non-smooth nonlinearities – piecewise stiffness and freeplay.

• Feedback linearisation for non-smooth nonlinearity.

• Multiple equilibria – zero and nonzero.

• Numerical demonstration.

• MODFLEX wing in preparation – multiple control surfaces, conventional motor-driven flaps, morphing using piezo-benders.
Selected Papers