VDM Based Damage Identification

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SUMMARY: New approach to the damage identification problem based on analysis of perturbation of elastic wave propagation is presented. The proposition is based on the use of pre-computed time dependent, dynamic influence matrix describing structural response to locally generated unit impulses. The global structural dynamic response can be decomposed on part caused by external excitation in undamaged structure and disturbance caused by the structural defects. Assuming possible locations of all potential defects in advance, an optimisation technique with analytically calculated gradients can be applied to solve the problem of the most probable defect locations. Theoretical background as well as numerical results are presented.

KEYWORDS: damage identification, inverse non-linear dynamic problem, dynamic sensitivity analysis.

INTRODUCTION

The damage detection systems based on array of piezoelectric transducers sending and receiving strain waves are intensively discussed by researchers recently (e.g. refs.1, 2). The signal-processing problem is the crucial point in this concept and a neural network based method is one of the most often suggested approaches to develop a numerically efficient solver for this problem (e.g. Ref.3).

The purpose of this chapter is to propose an alternative approach to the inverse dynamic analysis problem (after Ref.7). Generalising so called VDM (Virtual Distortion Method, Ref.4) approach on dynamic problems, a dynamic influence matrix D concept will be introduced. Pre-computing of the time dependent matrix D allows decomposition of the dynamic structural response on components caused by external excitation in undamaged structure (the linear part) and on components describing perturbations caused by the internal defects (the non-linear part). In the consequence, analytical formulae for calculation of these perturbations and the corresponding gradients can be derived. The physical meaning of so-called *virtual distortions* used in this paper can be explained with the notion of externally induced strains (non-compatible in general, e.g. caused by piezoelectric transducers, similarly to the effect of non-homogeneous heating). The compatible strains and self-equilibrated stresses are structural responses for these distortions.

Assuming possible locations of all potential defects in advance, an optimisation technique with analytically calculated gradients could be applied to solve the problem of the most probable defect locations. The considered damage can affect the local stiffness as well as the mass distribution modification. It is possible to identify the position as well as intensity of several, simultaneously generated defects. The proposed approach can be also applied to identification of multi-impact location and intensity.

Theoretical background as well as numerical results will be presented. This paper is a continuation of the problem described in Ref.6.

VDM BASED DESCRIPTION OF WAVE PROPAGATION

Let us describe the dynamic response of the strain increment $\Delta\varepsilon_A(t)$ in the location A and the time instance t as the superimposed response caused by impulses of so called *virtual distortions* increments $\Delta\varepsilon_\alpha^0(\tau)$ generated in the locations α and the time instances τ (cf. Fig.1):

$$\Delta\varepsilon_A(t) = \sum_{\tau\le t}\sum_{\alpha} D_{A\alpha}(t-\tau)\,\Delta\varepsilon_\alpha^0(\tau), \tag{1}$$

where the dynamic, time dependent, influence matrix $D_{A\alpha}(t-\tau)$ describes the corresponding dynamic response of the strain in location A and the time instance t, caused by the unit impulse virtual distortions forced in the locations α and time instances $\tau \le t$. Note that it is sufficient to compute only the matrix $D_{A\alpha}(t)$ which stores the response for the appropriate unit impulse distortion forced in the initial time instant $\tau = 0$. The virtual distortion increments $\Delta\varepsilon_\alpha^0(\tau)$ model excitations caused in locations α by the piezoelectric transducers (activated by an applied current increment). In the paper, we assume that small Greek subscripts (α) run through all locations of wave-generators while the capital Latin ones (A) run through locations of wave-receivers. The elements of the influence matrix $D_{A\alpha}(t)$ can be determined through the integration of the motion equations (e.g. using the Newmark's method) computed for the unit impulse excitation generated sequentially in the structural elements α. The unit impulse excitation can be supplied in form of initial velocity conditions: $v(0) = P\Delta t/m$, where P denotes, so called*, compensative* force corresponding to locally generated unit virtual distortion impulse $\varepsilon^0 = 1$, Δt is the integration time step, and m is the mass concentrated in the charged node of the loaded structural element α. Assuming (for simplicity of presentation) a discrete truss structure model (Fig.1), we can describe the transient function for the wave propagation generated in members $\alpha = 1,2$ and received in member $A = 20$. To this end it is necessary to determine, in advance, the time dependent dynamic influence matrix $D_{A\alpha}(t)$, where t runs through all time steps of the dynamic analysis: $t = \langle 0, T\rangle$. Having the influence matrix computed, we can calculate the superposition (1), where $\Delta\varepsilon_\alpha^0(\tau)$ describes (for the sequence of τ instances) the shape of the excited signal. Then, we can achieve the form of the strain in location A and the time period $\langle 0, T\rangle$ by summing the strain increments for all successive time instances $t \in \langle 0, T\rangle$:

$$\varepsilon_A(t) = \sum_{\tau\le t}\Delta\varepsilon_A(\tau) = \varepsilon_A(t-1) + \Delta\varepsilon_A(t). \tag{2}$$

In this way, the storage of the influence matrix $D_{A\alpha}(t)$, allows us to determine the transient function (between locations α and A) for any shape of the excited signal.

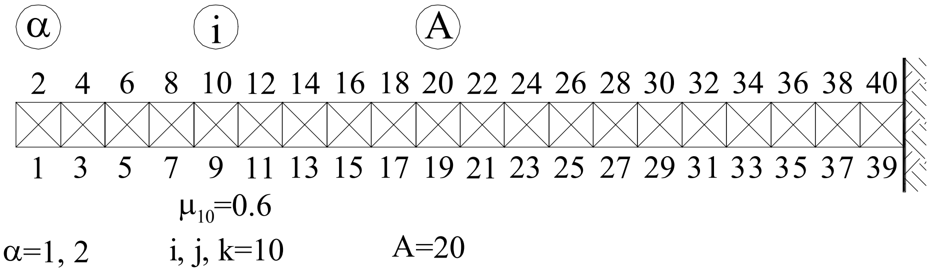

Fig.1: Truss-beam structure.

DAMAGE INFLUENCE DESCRIPTION

Let us apply the influence matrix based approach described above to the damage influence description. Three new, time-dependent, influence matrices ($D_{Ai}(t)$, $D_{i\alpha}(t)$, $D_{ij}(t)$) will be introduced. The method of computation of the matrices is similar to the one described in the previous chapter. In the case of any perturbation of elastic wave propagation caused by defects in structural members i (between the locations α of the wave generator and the location A of the wave observation), it is necessary to generalise the formula (1) adding the component $\Delta\varepsilon_A^R(t)$ related to the perturbation caused by these defects:

$$\Delta\varepsilon_A(t) = \Delta\varepsilon_A^L(t) + \Delta\varepsilon_A^R(t) = \sum_{\tau \le t}\left[\sum_{\alpha} D_{A\alpha}(t-\tau)\,\Delta\varepsilon_\alpha^0(\tau) + \sum_{i} D_{Ai}(t-\tau)\,\Delta\varepsilon_i^0(\tau)\right], \qquad (3)$$

where $\Delta\varepsilon_A^L(t)$ is the part of the strain increment caused by virtual distortion increments $\Delta\varepsilon_\alpha^0(t)$ modelling piezoelectric excitations, whereas $\Delta\varepsilon_A^R(t)$ is the component caused by virtual distortion increments $\Delta\varepsilon_i^0(t)$ simulating defects. From now on, we assume that small Latin indices (i, j, k, l) runs through all presumed locations of possible defects. The defect-simulating virtual distortion increment can be expressed by the following formula:

$$\Delta\varepsilon_i^0(t) = (1-\mu_i)\,\Delta\varepsilon_i(t), \qquad (4)$$

where $\varepsilon_i(t)$ denotes the strain in member i and the time instance t, while $\mu_i = E_i / E_i'$ denotes the ratio of the damaged member Young's modulus to the initial one. Therefore, the parameter $\mu_i \in \langle 0, 1\rangle$ specifies the size of the defect in location i (actually $\mu_i = 1$ means that there is no damage, while $\mu_i = 0$ means that the member i is completely damaged so that it can sustain no stresses). If we assume several possible-defect locations i (eventually, all structural elements of the structure), we can agree that vector μ_i specifies also the distribution of these defects.

The above relation (4) comes from the more general formula:

$$\mu_i = \frac{E_i}{E_i'} = \frac{\varepsilon_i(t) - \varepsilon_i^0(t)}{\varepsilon_i(t)} = \frac{\Delta\varepsilon_i(t) - \Delta\varepsilon_i^0(t)}{\Delta\varepsilon_i(t)}, \qquad (5)$$

which applies virtual distortions to simulate material parameter modifications (or material redistribution $\mu_i = A_i / A_i'$ etc.). Now, let us substitute the strains $\Delta\varepsilon_i(t)$ in the formula (4) through the formula analogous to equation (3):

$$\Delta\varepsilon_i(t) = \Delta\varepsilon_i^L(t) + \Delta\varepsilon_i^R(t) = \sum_{\tau \le t} \left[\sum_{\alpha} D_{i\alpha}(t-\tau)\, \Delta\varepsilon_\alpha^0(\tau) + \sum_{j} D_{ij}(t-\tau)\, \Delta\varepsilon_j^0(\tau) \right]. \qquad (6)$$

Here, similarly, increment $\Delta\varepsilon_i^L(t)$ is caused by the virtual distortion $\Delta\varepsilon_\alpha^0(t)$, modelling piezoelectric excitations, while increment $\Delta\varepsilon_i^R(t)$ is caused by the defect-simulating virtual distortions $\Delta\varepsilon_j^0(t)$. Now, the following relation between the defect parameters μ_i and the simulating this defect (in the time instance t) virtual distortion increment $\Delta\varepsilon_i^0(t)$ can be reached:

$$\sum_j \left[\delta_{ij} - (1-\mu_i)\, D_{ij}(0)\right] \Delta\varepsilon_j^0(t) = (1-\mu_i) \left[\sum_{\tau \le t} \sum_{\alpha} D_{i\alpha}(t-\tau)\, \Delta\varepsilon_\alpha^0(\tau) + \sum_{\tau < t} \sum_{j} D_{ij}(t-\tau)\, \Delta\varepsilon_j^0(\tau) \right]. \qquad (7)$$

Note that to achieve the above expression the following relation has been used:

$$\Delta\varepsilon_i^R(t) = \sum_{\tau \le t} \sum_{j} D_{ij}(t-\tau)\, \Delta\varepsilon_j^0(\tau) = \sum_j D_{ij}(0)\, \Delta\varepsilon_j^0(t) + \sum_{\tau < t} \sum_{j} D_{ij}(t-\tau)\, \Delta\varepsilon_j^0(\tau). \qquad (8)$$

For the distinguished time instant t, the formula (7) represents a set of i equations with $j = i$ unknowns $\Delta\varepsilon_j^0(t)$. To obtain $\Delta\varepsilon_j^0(t)$ for the entire time period $\langle 0, T \rangle$, we have to solve (step by step) the set (7) for all successive time instances $t \in \langle 0, T \rangle$. However, it is highly important (for the computation cost) to notice that in our algorithm $D_{ij}(0) = 0$. Considering this, the system of equations (7) should be given in the simple diagonal form:

$$\Delta\varepsilon_i^0(t) = (1-\mu_i) \left[\sum_{\tau \le t} \sum_{\alpha} D_{i\alpha}(t-\tau)\, \Delta\varepsilon_\alpha^0(\tau) + \sum_{\tau < t} \sum_{j} D_{ij}(t-\tau)\, \Delta\varepsilon_j^0(\tau) \right], \qquad (9)$$

which needs only computation of the right-hand side expressions. Knowing the defect parameters μ_i, the step by step (for the sequence of time instances t) determination of the increments $\Delta\varepsilon_i^0(t)$ can be performed making use of the formula (9). Then, knowing $\Delta\varepsilon_i^0(\tau)$ for $\tau \in \langle 0, t \rangle$, the strain increments in the observed location $\Delta\varepsilon_A(t)$ can be calculated making use of the equation (3). Summing these increments, like in the expression (2), we can determine the function of the strains $\varepsilon_A(t)$ in location A and the time period $\langle 0, T \rangle$.

SENSITIVITY ANALYSIS AND APPLICATION TO DAMAGE DETECTION

The partial derivatives $\partial\Delta\varepsilon_j^0 / \partial\mu_k$ can be determined from the following systems of equations obtained through differentiation of the formula (7):

$$\sum_j \left[\delta_{ij} - (1-\mu_i)\, D_{ij}(0)\right] \frac{\partial \Delta\varepsilon_j^0(t,\mu_l)}{\partial \mu_k} = -\delta_{ik}\, \Delta\varepsilon_i(t) + (1-\mu_i) \sum_{\tau<t} \sum_j D_{ij}(t-\tau)\, \frac{\partial \Delta\varepsilon_j^0(\tau,\mu_l)}{\partial \mu_k}. \quad (10)$$

Actually, for the distinguished time instant t, we have got here k sets of equations, where every set consists of i linear equations with j unknowns (and of course $i = j = k$). Finally, taking advantage of $D_{ij}(0) = 0$ (see comment to Eqs.(7), (9)) we can simplify the system to the following diagonal form:

$$\frac{\partial \Delta\varepsilon_i^0(t,\mu_l)}{\partial \mu_k} = -\delta_{ik}\, \Delta\varepsilon_i(t) + (1-\mu_i) \sum_{\tau<t} \sum_j D_{ij}(t-\tau)\, \frac{\partial \Delta\varepsilon_j^0(\tau,\mu_l)}{\partial \mu_k}. \quad (11)$$

Let us know apply the above sensitivity formulae to the inverse problem of damage identification requiring determination of the defect size and location (which are specified by the defect vector μ_i), knowing (from measurements) the functions of the strain response $\Delta\varepsilon_A^M(t)$ in locations A to the known excitation $\Delta\varepsilon_\alpha^0(\tau)$ generated in locations α. Therefore, the problem leads actually to the determination of the vector μ_i, where that assumed in advance locations i should allow every significant possibility of defect distribution. Assume for the objective function f the sum of the following measures f_A of the distance between the observed response $\varepsilon_A^M(t)$ in location A and the appropriate possible response $\varepsilon_A(t)$, which depends on the defect-simulating virtual distortions $\Delta\varepsilon_j^0(t,\mu_i)$:

$$f = \sum_A f_A = \sum_A \sum_t \left[d_A(t)\right]^2, \quad (12)$$

where

$$\begin{aligned} d_A(t) &= \varepsilon_A^M(t) - \varepsilon_A(t) = \varepsilon_A^M(t) - \left[\varepsilon_A^L(t) + \varepsilon_A^R(t)\right] = \varepsilon_A^M(t) - \sum_t \left[\Delta\varepsilon_A^L(t) + \Delta\varepsilon_A^R(t)\right] = \\ &= \varepsilon_A^M(t) - \sum_{\tau\le t} \sum_{\tau'\le\tau} \left[\sum_\alpha D_{A\alpha}(\tau-\tau')\, \Delta\varepsilon_\alpha^0(\tau') + \sum_j D_{Aj}(\tau-\tau')\, \Delta\varepsilon_j^0(\tau',\mu_i)\right]. \end{aligned} \quad (13)$$

The most probable defect identification leads to the minimisation problem $\min f$, with respect to the control parameters μ_i. To this end, the gradient approach can be applied, with the following analytical gradient calculated from the formulae (12), (13):

$$\frac{\partial f}{\partial \mu_k} = \sum_A \frac{\partial f_A}{\partial \mu_k} = -2 \sum_A \sum_t d_A(t) \left[\sum_{\tau\le t} \sum_{\tau'\le\tau} \sum_j D_{Aj}(\tau-\tau')\, \frac{\partial \Delta\varepsilon_j^0(\tau',\mu_i)}{\partial \mu_k}\right], \quad (14)$$

where the partial derivatives $\partial\Delta\varepsilon_j^0 / \partial\mu_k$ can be determined from Eq. (11).

The iterative algorithm for the multi-defect identification requires calculation (from Eqs.(9) and (11)) the defect-simulating distortion increments $\Delta\varepsilon_i^0(t)$ and their gradients $\partial\Delta\varepsilon_i^0 / \partial\mu_j$, for each time step of the dynamic analysis. Making use of these components, the objective function (12), (13) and its gradient (14) can be calculated. Heaving and the gradient of the objective function determined, a modification of the material redistribution can be proposed:

$$\mu_i = \mu_i - \frac{\partial f}{\partial \mu_i}\Delta, \qquad (15)$$

where the step length Δ can be adjusted e.g. due to the steepest descent optimisation strategy. Then, the calculation of the objective function and its gradient for the modified structure response can be performed in the next iteration. The cost of the initial computation is related to the determination of the structural dynamic responses for the unit impulses generated in all possible locations of the potential defects (the dynamic, time dependent influence matrix). Farther development of the proposed approach will be presented and applications to the damage detection will be discussed.

THE GENERAL CASE OF MASS AND/OR STIFFNESS MODIFICATIONS

Analogously to the formula (5), the simultaneous modifications of redistribution of local mass, as well as the stiffness, have to be taken into account through the following two, independent coefficients:

$$\mu_i^E = \frac{E_i}{E_i'} = \frac{\varepsilon_i(t) - \varepsilon_i^0(t)}{\varepsilon_i(t)}, \qquad \mu_i^M = \frac{M_i}{M_i'} = \frac{\ddot{\varepsilon}_i(t) - \ddot{\beta}_i^0(t)}{\ddot{\varepsilon}_i(t)}, \qquad (16)$$

described through two, independent virtual distortion fields $\varepsilon_i^0(t)$ and $\beta_i^0(t)$.

If we find these two fields satisfying (16), then the following two formulae describing dynamic responses of the modified structure:

$$M_{ki}\ddot{u}_i(t) + K_{ki}u_i(t) = F_k(t) \qquad (17)$$

(or equivalently described in the different form: $G_{kj}^T \widetilde{M}_{ji}\ddot{\varepsilon}_i(t) + G_{kj}^T E_{ji}\varepsilon_i(t) = F_k(t)$, where G_{kj} is the local geometric matrix and $\varepsilon_i = G_{il}u_l$) and the initial one, but affected by the virtual distortions simulating these modifications:

$$G_{kj}^T \widetilde{M}'_{ji}\left[\ddot{\varepsilon}_i(t) - \ddot{\beta}_i^0(t)\right] + G_{kj}^T E_{ji}\left[\varepsilon_i(t) - \varepsilon_i^0(t)\right] = F_k(t) \qquad (18)$$

lead to the same solution (in terms of $\varepsilon_i(t)$ and $\ddot{\varepsilon}_i(t)$). Then, subtracting equations (17) and (18) the following relation with vanishing (due to the satisfied conditions (16)) coefficients can be obtained. Therefore, the resultant structural displacements and accelerations also vanish, what confirms identity of the solutions for the problems (17) and (18).

In order to determine two virtual distortion fields simulating M and K modifications, the following relations expressed through the corresponding increments in each time step of dynamic analysis should be taken into account:

$$\Delta\varepsilon_i^0(t) = \left(1 - \mu_i^E\right)\Delta\varepsilon_i(t), \qquad \Delta\ddot{\beta}_i^0(t) = \left(1 - \mu_i^M\right)\Delta\ddot{\varepsilon}_i(t). \qquad (19)$$

Substituting the formula (cf. Eq.(16)):

$$\Delta\varepsilon_i(t) = \Delta\varepsilon_i^L(t) + \Delta\varepsilon_i^R(t) = \sum_{\tau \le t}\left\{\sum_{\alpha} D_{i\alpha}(t-\tau)\Delta\varepsilon_\alpha^0(\tau) + \sum_j D_{ij}(t-\tau)\left[\Delta\varepsilon_j^0(\tau) + \Delta\beta_j^0(\tau)\right]\right\} \qquad (20)$$

to $(19)_1$ and the following formula (Eq. (6) differentiated twice with respect to time):

$$\Delta\ddot{\varepsilon}_i(t)=\sum_{\tau\le t}\left\{\sum_{\alpha}\left[D_{i\alpha}(t-\tau)\Delta\ddot{\varepsilon}^0_\alpha(\tau)+2\dot{D}_{i\alpha}(t-\tau)\Delta\dot{\varepsilon}^0_\alpha(\tau)+\ddot{D}_{i\alpha}(t-\tau)\Delta\varepsilon^0_\alpha(\tau)\right]+\right.$$
$$\left.+\sum_j\left[D_{ij}(t-\tau)\left(\Delta\ddot{\varepsilon}^0_j(\tau)+\Delta\ddot{\beta}^0_j(\tau)\right)+2\dot{D}_{ij}(t-\tau)\left(\Delta\dot{\varepsilon}^0_j(\tau)+\Delta\dot{\beta}^0_j(\tau)\right)+\ddot{D}_{ij}(t-\tau)\left(\Delta\varepsilon^0_j(\tau)+\Delta\beta^0_j(\tau)\right)\right]\right\} \tag{21}$$

to $(19)_2$, the following two relations allowing determination of the increments $\Delta\varepsilon^0_i$, $\Delta\ddot{\beta}^0_i$ in each time step can be reached:

$$\Delta\varepsilon^0_i(t)=\left(1-\mu^E_i\right)\sum_{\tau<t}\left\{\sum_{\alpha}D_{i\alpha}(t-\tau)\Delta\varepsilon^0_a(\tau)+\sum_j D_{ij}(t-\tau)\left[\Delta\varepsilon^0_j(\tau)+\Delta\beta^0_j(\tau)\right]\right\},$$
$$\Delta\ddot{\beta}^0_i(t)=\left(1-\mu^M_i\right)\sum_{\tau<t}\left\{\sum_{\alpha}\left[D_{i\alpha}(t-\tau)\Delta\ddot{\varepsilon}^0_\alpha(\tau)+2\dot{D}_{i\alpha}(t-\tau)\Delta\dot{\varepsilon}^0_\alpha(\tau)+\ddot{D}_{i\alpha}(t-\tau)\Delta\varepsilon^0_\alpha(\tau)\right]+\right. \tag{22}$$
$$\left.+\sum_j\left[D_{ij}(t-\tau)\left(\Delta\varepsilon^0_j(\tau)+\Delta\beta^0_j(\tau)\right)+2\dot{D}_{ij}(t-\tau)\left(\Delta\dot{\varepsilon}^0_j(\tau)+\Delta\beta^0_j(\tau)\right)+\ddot{D}_{ij}(t-\tau)\left(\Delta\varepsilon^0_j(\tau)+\Delta\beta^0_j(\tau)\right)\right]\right\}.$$

Note that the property $D_{ij}(0)=0$ has been taken into account in the above relations.

Making use of the formulae (22), the algorithm for determination of the two virtual distortion fields $\Delta\varepsilon^0_i$, $\Delta\ddot{\beta}^0_i$ can be perform as follows: for each time step t calculate $\Delta\varepsilon^0_i$ and $\Delta\ddot{\beta}^0_i$ from (22) and then successively compute:

$$\Delta\ddot{\varepsilon}^0_i(t)=\frac{\Delta\varepsilon^0_i(t)-\Delta\varepsilon^0_i(t-1)}{\Delta t},\qquad \Delta\ddot{\varepsilon}^0_i(t)=\frac{\Delta\dot{\varepsilon}^0_i(t)-\Delta\dot{\varepsilon}^0_i(t-1)}{\Delta t},$$
$$\Delta\dot{\beta}^0_i(t)=\Delta\ddot{\beta}^0_i(t)\Delta t+\Delta\dot{\beta}^0_i(t-1),\quad \Delta\dot{\beta}^0_i(t)=\Delta\ddot{\beta}^0_i(t)\Delta t+\Delta\dot{\beta}^0_i(t-1) \tag{23}$$

to have components for calculation of $(22)_2$ in the next time step.

Knowing the defect parameters μ^E_i, μ^M_i the step by step (for the sequence of time instances t) determination of the increments $\Delta\varepsilon^0_i(t)$, $\Delta\ddot{\beta}^0_i(t)$ can be performed making use of the formula (22). Then, knowing $\Delta\varepsilon^0_i(\tau)$ and $\Delta\ddot{\beta}^0_i(\tau)$ for $\tau\in\langle 0,t\rangle$, the strain increments in the observed location $\Delta\varepsilon_A(t)$ can be calculated making use of the equation analogous to (3). Summing these increments, like in the expression (2), we can determine the function of the strains $\varepsilon_A(t)$ in location A and the time period $\langle 0,T\rangle$.

The damage identification, corresponding to both modifications M and K can be done analogously to the procedure descried above.

TESTING EXAMPLE

The truss cantilever beam model (Fig.1) has been used to verify operation of the proposed VDM based sensitivity analysis and the damage identification technique. It has been assumed that the piezoelectric transducers generating sinusoidal shape of excitation are located in members 1 and 2 (simultaneous extension of the same intensity but the opposite sign are generated). The piezoelectric sensor observing wave propagation has been located in member 20. Then, two simulation processes of wave propagation have been performed. The first one, for the initial (undamaged) structural configuration and the second for the structure with damaged members: 8, 9, 10, 11 and 12 (the Young's modulus reduced by 40%, 20%, 30%, 20% and 10%, up to $\mu_8 = 0.6$, $\mu_9 = 0.8$, $\mu_{10} = 0.7$, $\mu_{11} = 0.8$ and $\mu_{12} = 0.9$, respectively). The corresponding results ($\varepsilon_{20}(t) / \left(\frac{P_0}{EA}\right)$, where EA defines stiffness of member 20) are presented in Fig.2, where 200 time steps have been applied.

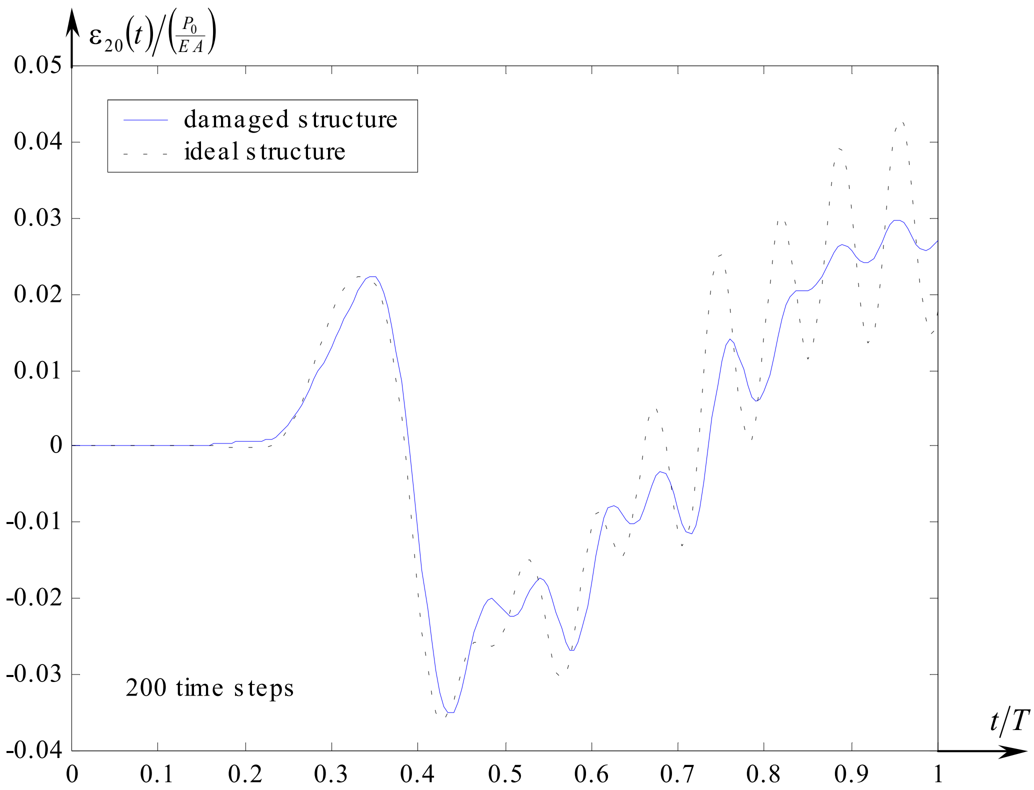

Fig. 2: Responses of ideal and damaged structure.

Applying this second result (for $\mu_8 = 0.6$, $\mu_9 = 0.8$, $\mu_{10} = 0.7$, $\mu_{11} = 0.8$ and $\mu_{12} = 0.9$) as the measured response $\varepsilon_{20}^{M}(t)$, the damage identification algorithm (with five control parameters μ_8, μ_9, μ_{10}, μ_{11} and μ_{12}) has been performed. The corresponding results (Figs.3, 4) demonstrate the iterative process of damage identification. It is shown that during the process, the objective function and its gradient components converge to zero (Fig.3), while the defect parameters aim to the correct solution: $\mu_8 = 0.6$, $\mu_9 = 0.8$, $\mu_{10} = 0.7$, $\mu_{11} = 0.8$ and $\mu_{12} = 0.9$ (Fig.4).

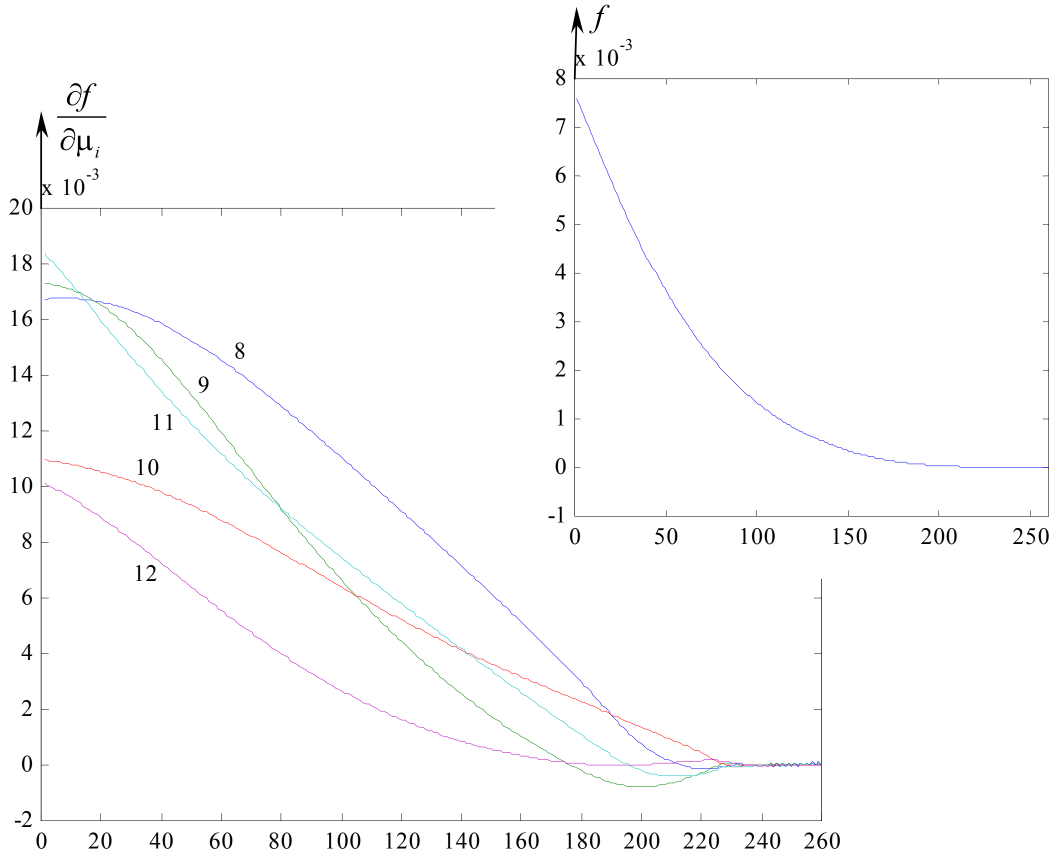

Fig. 3: Damage identification process – the objective function and its gradient.

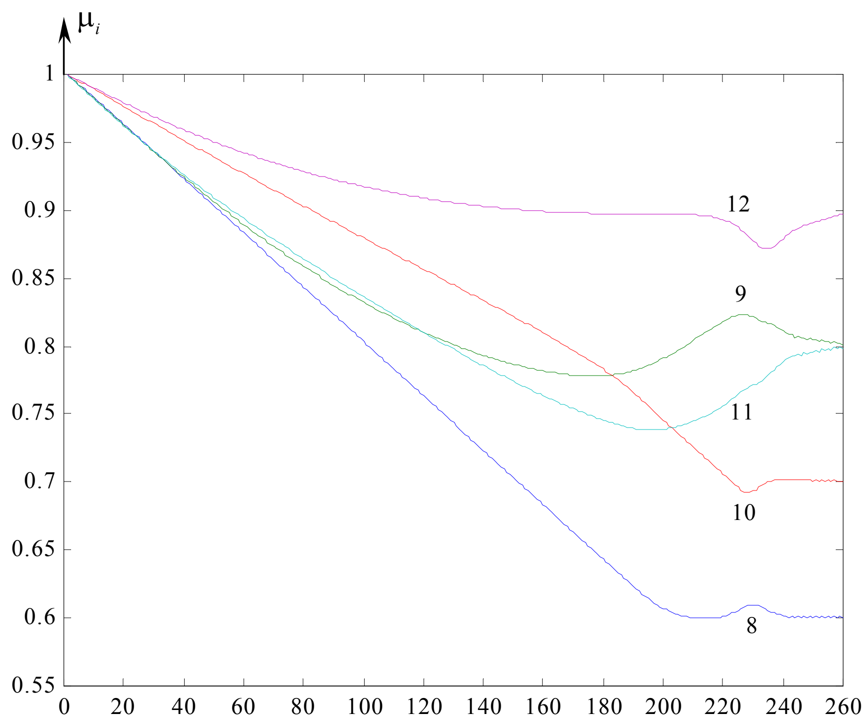

Fig. 4: Damage identification process – the defect parameters.

CONCLUSIONS

- New approach to the damage identification problem based on analysis of perturbation of elastic wave propagation has been presented.

- The proposition is based on the use of pre-computed time dependent, dynamic influence matrix describing structural response to locally generated unit impulses. The global structural dynamic response can be decomposed on the following two parts: the first one, caused by external excitation in undamaged structure and the second (perturbing) one, caused by the structural defects (modelled through so called *virtual distortions* multiplied by the influence matrix).

- This VDM (Virtual Distortion Method) based formulation allows numerically efficient, analytical gradient calculation (with respect to local defect/virtual distortion intensity)

- Assuming possible locations of all potential defects in advance, an optimisation technique with analytically calculated gradients has been applied to solve the problem of the most probable defect locations

- The proposed numerical tool for the inverse non-linear, dynamic problem analysis has been tested on the truss beam structure, identifying accurately five simultaneous defects with different intensities

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